86. On the Yukawa-coupled Klein-Gordon-Schrödinger Equations in Three Space Dimensions

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(Comm. by Kinjirô KUNUGI, M. J. A., June 3, 1975)

1. Introduction and notation. We consider the Yukawa-coupled Klein-Gordon-Schrödinger equations in R^3:

$$(1)\qquad \begin{aligned} &i\frac{\partial\psi(t,x)}{\partial t}+\Delta\psi(t,x)=g\psi(t,x)\phi(t,x),\\ &\left(\Delta-\frac{\partial^2}{\partial t^2}-\mu^2\right)\phi(t,x)=g\psi(t,x)\overline{\psi(t,x)}, \end{aligned}$$

which represent the classical model of dynamics of conserved complex nucleon fields ψ interacting with neutral real scalar meson fields ϕ. The constant μ describes mass of a meson and g a coupling real constant.

In the case of one space dimension, the existence of global solutions of the Cauchy problem has been established by the authors [3]. In the case of relativistic fields, that is, when nucleons are governed by the Dirac spinor fields, we must treat the coupled Klein-Gordon-Dirac equations:

$$\left(i\gamma_\nu\frac{\partial}{\partial x_\nu}-m\right)\psi=g\psi\phi \qquad \left(\frac{\partial}{\partial x_0}=\frac{\partial}{\partial t}\right),$$
$$\left(\Delta-\frac{\partial^2}{\partial t^2}-\mu^2\right)\phi=g\psi\bar{\psi},$$

which were investigated by Chadam and Glassey [1], [2].

In this paper, our purpose is to state the existence and uniqueness theorems for global solutions of the initial-boundary value problem for the system (1) in Ω with boundary conditions:

$$(2)\qquad \psi(t,x)=\phi(t,x)=0 \quad \text{for } x\in\partial\Omega \quad\text{and}\quad t\geqslant 0$$

and initial conditions:

$$(3)\qquad \psi(0,x)=\psi_0(x),\ \phi(0,x)=\phi_0(x) \quad\text{and}\quad \phi_t(0,x)=\phi_1(x) \quad \text{for } x\in\Omega,$$

where Ω denotes a bounded domain in R^3 with sufficiently smooth boundary $\partial\Omega$.

In section 2, we refer to the global existence theorem of the initial-boundary value problem (1)–(3), and the main tool for proving them. In section 3, we represent the uniqueness result. In section 4, we investigate the regularity properties of solutions of (1)–(3).

In this note, we state the results only. Detailed proofs will be