83. *Central Class Numbers in Central Class Field Towers*

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1. Introduction. Let $K_0=k$ be an algebraic number field of finite degree and K_n be the central class field of K_{n-1} over k, i.e. the maximal unramified abelian extension over K_{n-1} such that the Galois group of K_n over K_{n-1} is contained in the center of the Galois group of K_n over k. Then the sequence of fields

$$k=K_0\subseteq K_1\subseteq\cdots K_{n-1}\subseteq K_n\subseteq\cdots$$

is called the central class field tower of k, and the extension degree $z_n=[K_{n+1}:K_n]$ is called the central class number[1] of K_n over k. $z_0=[K_1:k]$ is the class number of k.

The existence of algebraic number fields admitting infinite central class field towers is shown by Golod and Šafarevič [5]. In connection with the result, Brumer [2], Furuta [4] and Roquette [7] estimate lower bounds on the l-rank of the ideal class group of a finite Galois extension, where l is a rational prime.

The aim of the present paper is to give an upper bound on the central class number z_n of K_n over k (Main Theorem) and also to give an upper bound on the rank of the Galois group of K_{n+1} over K_n (Theorem 5).

Main Theorem. *Let z_n be as above and d be the minimal number of generators of the ideal class group of k. Then we have*

$$z_{n-1}^d\equiv 0 \pmod{z_n} \qquad \text{for } n>1$$

and

$$z_0^{z_0(d-1)}\equiv 0 \pmod{z_1} \qquad \text{for } n=1.$$

In particular,

$$h^{h(d-1)d^{n-1}}\equiv 0 \pmod{z_n} \qquad \text{for } n\geqq 1,$$

where $h=z_0$ is the class number of k.

2. Notation. Throughout this paper the following notation will be used.

Z	the ring of rational integers
Q	the field of rational numbers
K^*	the multiplicative group of all non-zero elements of a field K
J_K	the idele group of a finite algebraic number field K

1) Cf. Furuta [3].