81. A Note on Isolated Singularity. II

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0. Introduction. This is a brief résumé of the second half of the study whose first part has already been announced [3]. The main purpose is to investigate the structure of an isolated singularity when it admits a C^*-action, especially, to obtain some formula concerning the characters of the representations of C^* over various cohomology groups associated with the singularity.

1. Basic concepts. A C^*-action over an isolated singularity (X, x) is a family $T(c), c \in C^*$ of analytic homeomorphisms of X onto itself satisfying that $T(c)x=x$, $T(cc')=T(c)T(c')(c, c' \in C^*)$, and that $T: X\times C \ni (x, c)\to T(z)c \in X$ is analytic. Throughout this note we assume that the constants are the only invariant elements of $\Omega^0_{X,x}$ under this action. Let ξ be the generating vector field of this action. The interior multiplication $i(\xi)$ is an anti-derivation of $\Omega_X^{\cdot}$ regarded as the sheaf of graded algebra. It is well known that the Poincaré complex $\Omega_X^{\cdot}$ is acyclic in this case. However we have some more

Lemma 1. *Under the above condition the sequences*

$$\cdots \xrightarrow{d} \mathcal{H}^0_x(\Omega^p_X) \xrightarrow{d} \mathcal{H}^0_x(\Omega^{p+1}_X) \xrightarrow{d} \cdots$$

$$\cdots \longrightarrow \Omega^p_X \xrightarrow{i(\xi)} \Omega^{p-1}_X \xrightarrow{i(\xi)} \cdots \xrightarrow{i(\xi)} \Omega^0_X \xrightarrow{\alpha} (\iota_x)_* C \longrightarrow 0$$

are exact, where ι_x denotes the inclusion $x \hookrightarrow X$ and α the average map $\Omega^0_{X,x} \ni f \to \int_0^1 T(e^{2\pi i\theta})^* f d\theta \in (\iota_x)_* C_x$.

If we set $\Omega^p_\xi = i(\xi)\Omega^{p+1}_X$, then we have the short exact sequences $0\to\Omega^p_\xi\to\Omega^p_X\to\Omega^{p-1}_\xi\to 0$. From these we obtain the following Gysin type sequences

$$0 \longrightarrow \mathcal{H}^0_x(\Omega^p_\xi) \longrightarrow \mathcal{H}^0_x(\Omega^p_X) \longrightarrow \mathcal{H}^0_x(\Omega^{p-1}_\xi) \longrightarrow \cdots$$

$$\cdots \longrightarrow \mathcal{H}^q_x(\Omega^p_\xi) \longrightarrow \mathcal{H}^q_x(\Omega^p_x) \longrightarrow \mathcal{H}^q_x(\Omega^{p-1}_\xi) \longrightarrow \cdots.$$

Using these, we can prove

Theorem 1. *Let the notation and the assumption be as above. Assume that (X, x) satisfies the condition* (L). *Then $\mathcal{H}^q_x(\Omega^p_\xi)=0$ for (p, q) such that $p+q\neq \dim X$, $q\neq p+1$, $q<\dim X$, and there are natural isomorphisms $\mathcal{H}^q_x(\Omega^p_X)\simeq \mathcal{H}^q_x(\Omega^{p+1}_X)$ for (p, q) such that $p+q=\dim X$, $0<q<\dim X$.*

Remark. If $\dim X$ is even, the proof requires some technique from Kähler geometry, though we can avoid the use of this in case