78. *On Deformations of the Calabi-Eckmann Manifolds*

By Kazuo AKAO
University of Tokyo

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1. The purpose of this note is to give an explicit construction of an effectively parametrized and complete family of deformations of usual Calabi-Eckmann manifolds.

E. Calabi and B. Eckmann [1] constructed a class of simply connected non-Kähler complex manifolds which are homeomorphic to the product of two odd-dimensional spheres $S^{2n+1}\times S^{2m+1}$. They are the simplest examples of so-called non-Kähler C-manifolds, i.e., simply connected compact complex homogeneous spaces. Deformations of compact complex homogeneous spaces have been studied by several authors. First Kodaira-Spencer [4] constructed an effectively parametrized and complete family of deformations of Hopf surfaces. M. Ise [3] and P. A. Griffiths [2] calculated $H^{\nu}(M,\Theta)$ for C-manifolds M. The latter proved that for each abelian Lie subalgebra of $H^1(M,\Theta)$, there exists a family of deformations of M corresponding to it by the Kodaira-Spencer map. However his method was quite implicit, using similar methods as in Kuranishi's proof of the existence of versal deformations and, in particular, does not give sufficient informations on deformed structures. In the sequel we shall construct an effectively parametrized and complete family of deformations of the Calabi-Eckmann manifolds, using Tits' construction of homogeneous spaces, and state some properties of deformed structures. It seems that our method may be applied to construction of families of deformations of more general non-Kähler C-manifolds. In what follows Θ denotes the sheaf of germs of holomorphic vector fields, and I_n the unit matrix in $GL(n,\boldsymbol{C})$.

2. Tits' construction of the Calabi-Eckmann manifolds. We define a Calabi-Eckmann manifold, following J. Tits [5]. For each $t\in\boldsymbol{C}$, let g_t^{λ} be the biholomorphic automorphism of $\boldsymbol{C}^p-(0)\times\boldsymbol{C}^q-(0)$ which maps (z,w) to $(z\exp t, w\exp\lambda t)$, where z and w are, resp., the standard coordinates of $\boldsymbol{C}^p$ and $\boldsymbol{C}^q$, and λ is a fixed complex number with $\operatorname{Im}\lambda\neq 0$. Let G_{λ} be the one-parameter complex Lie group consisting of g_t^{λ}'s. G_{λ} operates freely and properly on $\boldsymbol{C}^p-(0)\times\boldsymbol{C}^q-(0)$, hence we can construct the quotient manifold $M=\boldsymbol{C}^p-(0)\times\boldsymbol{C}^q-(0)/G_{\lambda}$. M is called a Calabi-Eckmann manifold. The natural projection from $\boldsymbol{C}^p-(0)\times\boldsymbol{C}^q-(0)$ to $\boldsymbol{P}^{p-1}\times\boldsymbol{P}^{q-1}$ makes M a complex analytic fibre bundle