77. Notes on Complex Lie Semigroups

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1. By a complex space, we mean a reduced, Hausdorff, complex analytic space. A semigroup S is called *a complex Lie semigroup* if and only if (1) S is a complex space and (2) the product $(x, y) \in S \times S \rightarrow xy \in S$ is a holomorphic map.

Important examples of complex Lie semigroups are: (1) a (finite dimensional, associative) C-algebra with respect to its product, (2) End(G)=the set of all (holomorphic) endomorphisms of a connected complex Lie group G (c.f., Chevalley [1]) and (3) Hol(V, V)=the set of all holomorphic maps of a compact complex space V into itself (Douady [2]).

The purpose of this note is to state some results on the structures of complex Lie semigroups with 1 (the identity) or 0 (zero). Details will be published elsewhere.

2. By *a subsemigroup* (resp. *an ideal*) *of a complex Lie semigroup*, we mean a subsemigroup (resp. an ideal) in the usual sense which is at the same time a closed complex subvariety. By *an isomorphism of complex Lie semigroups*, we mean an isomorphism as semigroups which is at the same time a biholomorphic map.

3. We first state the following Theorems 1 and 2.

Theorem 1. *Let S be a complex Lie semigroup with* 1 (*the identity*). *We denote by $G(S)$ the set of all invertible elements of S. Then* (1) *$G(S)$ is a non-singular open subspace of S and is a complex Lie group with respect to the product in S,* (2) *the closure $\overline{G(S)}$ is a union of some irreducible components of S and is a subsemigroup of S and* (3) *$\overline{G(S)}-G(S)$ is an ideal of $\overline{G(S)}$.*

Corollary. *Let V be a compact complex space. Then* Aut(V) (*the group of all biholomorphic maps of V onto itself*) *is* (*open and*) *closed in* Hol(V, V) *with respect to the compact-open topology.*

Theorem 2. *Let S be a complex Lie semigroup with* 1. *Assume that S is irreducible as a complex space. Then* (1) *the set of all singular points of an ideal of S is also an ideal of S,* (2) *each irreducible component of an ideal of S is also an ideal of S and* (3) *any ideal of S is written as a finite union of ideals of S which are irreducible as complex spaces.*

Now, let S be a complex Lie semigroup with 0 (zero). Locally, we