119. *Theory of H-valued Fourier Hyperfunctions*

By Yoshifumi ITO*) and Shigeaki NAGAMACHI**)

(Comm. by Kinjirô KUNUGI, M. J. A., Sept. 12, 1975)

§ 0. Recently the theory of vector valued hyperfunctions has been developed by Ion, P.D.F. and T. Kawai [1]. It has been done by the method of 'soft analysis' in parallel with Sato's theory of hyperfunctions (see Sato, M. [8]). In this paper, we construct the theory of vector valued Fourier hyperfunctions by the method analogous to Kawai's method of constructing the theory of Fourier hyperfunctions (see Kawai, T. [3], [4]). It is known that this theory of vector valued Fourier hyperfunctions is useful in its applications to some problems in the quantum field theory (see Nagamachi, S. and N. Mugibayashi [7]).

We construct the sheaf ${}^H\mathcal{R}$ of H-valued Fourier hyperfunctions over $\boldsymbol{D}^n$ as the n-th derived sheaf of ${}^H\tilde{\mathcal{O}}$ with support in $\boldsymbol{D}^n$, where $\boldsymbol{D}^n$ is the radial compactification of $\boldsymbol{R}^n$ (see Kawai, T. [4]) and H is a separable complex Hilbert space and ${}^H\tilde{\mathcal{O}}$ is the sheaf of slowly increasing H-valued holomorphic functions over $\boldsymbol{D}^n\times\sqrt{-1}\boldsymbol{R}^n$ (see Ito, Y. and S. Nagamachi [2]).

Next we realize H-valued Fourier hyperfunctions with supports in a compact set K in $\boldsymbol{D}^n$ as continuous linear operators from $\underset{\sim}{\mathcal{O}}(K)$ to H (as to $\underset{\sim}{\mathcal{O}}(K)$, see Kawai, T. [4]). Namely, we show that the space $H^n_K(V, {}^H\tilde{\mathcal{O}})$ of H-valued Fourier hyperfunctions with supports in K is isomorphic to the space $L_b(\underset{\sim}{\mathcal{O}}(K); H)$ of all continuous linear operators from $\underset{\sim}{\mathcal{O}}(K)$ to H equipped with the topology of bounded convergence. We also show that the space $H^n_K(V, {}^H\tilde{\mathcal{O}})$ is isomorphic to the tensor product $H^n_K(V, \tilde{\mathcal{O}})\hat{\otimes}H$ of the space $H^n_K(V, \tilde{\mathcal{O}})$ of scalar valued Fourier hyperfunctions with supports in K and the Hilbert space H. These facts are very interesting in comparison with the fact that the spaces of some kinds of vector valued functions and the spaces of vector valued distributions introduced by L. Schwartz have the same properties.

The sheaf ${}^H\mathcal{R}$ is a flabby sheaf and its restriction to $\boldsymbol{R}^n$ coincides with the sheaf ${}^H\mathcal{B}$ of H-valued hyperfunctions over $\boldsymbol{R}^n$, and its global sections are stable under Fourier transformations. Hence any H-valued hyperfunction on $\boldsymbol{R}^n$ can be extended to an H-valued Fourier

*) Department of Mathematics, College of General Education, Tokushima University, Tokushima.

**) Department of Mathematics, Faculty of Engineering, Tokushima University, Tokushima.