115. On almost Primes in Arithmetic Progressions. II

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§ 1. Let P_r denote as usual a number which has at most r prime factors counting multiplicities. In our previous paper [2] we have proved that there are numbers such that

$$P_2 \ll k^{11/10}, \qquad P_2 \equiv l \pmod{k},$$
$$P_3 \ll k(\log k)^{70}, \qquad P_3 \equiv l \pmod{k},$$

for almost all reduced residue classes $l \pmod{k}$. The purpose of the present note is to study briefly the dual problem in which the reduced residue class l is fixed and the modulus k runs over certain interval. We prove

Theorem. *Let l be a fixed non-zero integer. Then there is a P_3 such that*

$$P_3 \ll k(\log k)^{70}, \qquad P_3 \equiv l \pmod{k},$$

for almost all k, $(k,l)=1$.

Our proof depends on two recent results: one from [2] which concerns to a compact presentation of the sieve procedure of Jurkat and Richert, and the other from [1] which is a simple variant of the dispersion method of Linnik. These are embodied in lemmas of the next paragraph.

Notations. In what follows we always have $(k,l)=1$, and we may assume that l is a positive integer. x is a positive and sufficiently large parameter. $\varphi(n)$ denotes the Euler function, and $d(n)$, $d_5(n)$ are divisor functions. (n,m) and $[n,m]$ denote the greatest common divisor and the least common multiple between n and m, respectively.

§ 2. Let $z \geqq 2$ be arbitrary, and let

$$P_k(z) = \prod_{\substack{p \leqq z \\ p \nmid k}} p, \qquad \Gamma_k(z) = \prod_{\substack{p \leqq z \\ p \nmid k}} \left(1 - \frac{1}{p}\right),$$

p being generally a prime number. We introduce another parameter w such that $z \leqq w$, and we put, for any non-negative constant ζ,

$$V_\zeta(x;k,l;z,w) = \sum_{\substack{n \equiv l \ (\mathrm{mod}\ k) \\ n \leqq x \\ (n,l)=1 \\ (n, P_k(z))=1}} \left\{1 - \zeta \sum_{\substack{p \mid n \\ p \nmid kl \\ z \leqq p < w}} \left(1 - \frac{\log p}{\log w}\right)\right\},$$

$$S(x;k,l;z,w) = \sum_{\substack{n \equiv l \ (\mathrm{mod}\ k) \\ n \leqq x \\ (n,l)=1}} \sum_{\substack{p^2 \mid n \\ p \nmid kl \\ z \leqq p < w}} 1.$$