114. *Asymptotic Equivalence in a Dynamical System*

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1. Introduction. Let X be a metric space with its metric d. A dynamical system on X [1, p. 5] is defined to be an ordered triple (X, R, π) consisting of X, the real line R and a map $\pi: X\times R\to X$ such that:

(a) $\pi(x, 0)=x$ for any $x\in X$,

(b) $\pi(\pi(x, s), t)=\pi(x, s+t)$ for any $x\in X$ and all $s, t\in R$,

(c) π is continuous on $X\times R$.

Given a dynamical system on X, the space X is called the phase space of the dynamical system.

An equivalence relation J on (X, R, π) is said to be invariant if $(x, y)\in J$ implies $(\pi(x, t), \pi(y, t))\in J$ for any $t\in R$.

A large amount of research of the invariant equivalence relations in the phase space of dynamical systems has been done (e.g., see [2], [3], or [4]). However, the main concern of these is either the case in which the invariant equivalence relation is closed, or the case in which the phase space is compact.

In this paper we introduce an invariant equivalence relation, i.e., "*asymptotic equivalence*", which is neither closed nor the phase space compact, and then investigate the possibility of the construction of the quotient dynamical system induced by the equivalence relation. Main results obtained are Theorem 3.3 (a necessary and sufficient condition for the canonical surjection to be open) and Theorem 3.5 which gives a necessary and sufficient condition for the phase space of the quotient dynamical system to be Hausdorff.

2. Asymptotic equivalence.

Definition 2.1. If

$$d(\pi(x, t), \pi(y, t))\to 0 \qquad (t\to+\infty),$$

then x is said to be *asymptotically equivalent* to y, which is denoted xAy.

Remark 2.2. It is clear that the asymptotic equivalence A on (X, R, π) is an invariant equivalence relation.

Remark 2.3. The asymptotic equivalence A on (X, R, π) is not a closed relation, since we have a counterexample [1, p. 68, 2.4]: a dynamical system defined on R^2 by the differential equations