109. On the Global Existence of Solutions of Differential Equations on Closed Subsets of a Banach Space

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(Comm. by Kôsaku YOSIDA, M. J. A., Sept. 12, 1975)

1. Introduction. Let D be a subset of a real Banach space X and A be a continuous function from $[0, +\infty)\times D$ into X. In this paper we consider the initial value problem

(IVP) $$u'=A(t,u), \qquad u(0)=x,$$

where x is given in D. By a solution of (IVP) or of (IVP; x), we mean a continuously differentiable function u from $[0, +\infty)$ into D such that $u(0)=x$ and $u'(t)=A(t, u(t))$ for all $t\geqq 0$.

This kind of problem has been treated by many authors; for example, see Crandall [1], Lovelady-Martin [3], Martin [4], Pavel [5], [6] and the cited papers in them.

The purpose of this paper is to establish a global existence theorem for (IVP) under some conditions which are similar to those treated in [4] but somewhat weaker than them. Our theorem gives some simplifications and improvements of results in [4] and also provides an answer to a question raised by Martin [4].

2. Existence theorem. Let X be a real Banach space, X^* the dual space of X and denote by $\langle x, f\rangle$ the natural pairing between $x\in X$ and $f\in X^*$. For each $x, y\in X$, define

$$\langle y, x\rangle_i=\inf\{\langle y, f\rangle ; f\in F(x)\},$$

where F is the duality mapping from X into X^*, i.e., F is defined by

$$F(x)=\{f\in X^* ; \langle x, f\rangle=\|x\|^2=\|f\|^2\}$$

for each $x\in X$.

Now, let D be a closed subset of X, A a function from $[0, +\infty)\times D$ into X and consider the following conditions:

(A1) A is continuous from $[0, +\infty)\times D$ into X;

(A2) there is a real-valued continuous function ω defined on $[0, +\infty)$ such that

$$\langle A(t,x)-A(t,y), x-y\rangle_i\leqq \omega(t)\,\|x-y\|^2$$

for all (t, x) and (t, y) in $[0, +\infty)\times D$;

(A3) $\liminf_{h\to 0+} h^{-1}d(x+hA(t,x), D)=0$ for each (t, x) in $[0, +\infty)\times D$, where $d(z, D)$ stands for the distance from $z\in X$ to D.

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