# 155. On the Fundamental Solution of a Degenerate Parabolic System

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**Introduction.** In the recent paper [2], the author has shown that the method used in C. Tsutsumi [3] to construct the pseudo-differential symbol of the fundamental solution for a degenerate parabolic pseudo-differential operator is applicable to some parabolic systems. The purpose of the present paper is to show that the above method is also applicable to a parabolic system which degenerates at $t=0$. As an application we construct in § 2 the pseudo-differential symbol of the fundamental solution of a degenerate parabolic operator of higher order which includes the operator treated by M. Miyake [1]. In the following the notation of [2] will be freely used.

**1. The fundamental solution of a degenerate system.** Let us consider the Cauchy problem for a system of pseudo-differential equations

$$\partial_t u(t,x)+p(t;X,D_x)u(t,x)=0, \tag{1}$$

$$\lim_{s\searrow 0} u(t,u)=u_0(x), \tag{2}$$

where $p(t;x,\xi)\in\mathcal{E}_t^0(S^m_{\rho,\delta})$, $0\leq\delta<\rho\leq 1$. We denote by $z(t,s;x,\xi)$ an $M\times M$ matrix of symbols which satisfies $\partial_t z(t,s;x,\xi)+p(t;x,\xi)z(t,s;x,\xi)=0$, $z(s,s;x,\xi)=I$, where $I$ denotes the identity matrix. We denote by $|p|$ the norm of an $M\times M$ matrix $p$, i.e., $p=\sup\{|py|/|y|;0\neq y\in\boldsymbol{C}^M\}$.

**Definition.** We say that a system of pseudo-differential operators $\partial_t+p(t;X,D_x)$ satisfies the property $(F)$, when there exists a non-negative continuous function $\lambda(t;x,\xi)$ and following two conditions are satisfied:

i) For any $\alpha,\beta$ there exists a constant $C_{\alpha,\beta}$ such that

$$\int_s^t |p^{(\alpha)}_{(\beta)}(\sigma;x,\xi)|\,d\sigma\leq C_{\alpha,\beta}\langle\xi\rangle^{-\rho|\alpha|+\delta|\beta|}\left\{\int_s^t \lambda(\sigma;x,\xi)d\sigma+1\right\} \quad \text{for } 0\leq s\leq t\leq T. \tag{3}$$

ii) There exist constants $d>0$ and $C>0$ such that

$$|z(t,s;x,\xi)|\leq C\exp\left[-d\int_s^t \lambda(\sigma;x,\xi)d\sigma\right] \qquad \text{for } 0\leq s\leq t\leq T. \tag{4}$$

When a system $\partial_t+p(t;X,D_x)$ is parabolic in the sense of Petrowskii, it satisfies the property $(F)$ with $\lambda(t;x,\xi)=\langle\xi\rangle^m$ in any finite layer $[0,T]\times R^{2n}_{x,\xi}$. But in the case of degenerate $p(t;x,\xi)$, we must choose a degenerate $\lambda(t;x,\xi)$. Here we give a class of systems for which the property $(F)$ is easily verified.