145. *Eisenstein Integrals and Singular Cauchy Problems*

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1. The classical Euler-Poisson-Darboux (EPD) equations of Weinstein (see e.g. [15]), and various formulas arising in their solution, are known to possess group theoretic content, and various other analogous classes of singular Cauchy problems also have been studied from this point of view (cf. [4]–[6], [11]). We will discuss here some aspects of the general situation in the context of harmonic analysis on symmetric spaces (cf. [7]–[10], [12]–[14] for notation). Thus let G be a real connected noncompact semisimple Lie group with finite center and K a maximal compact subgroup so that $V=G/K$ is a symmetric space of noncompact type. Let $g^{\sim}=k^{\sim}+p$ be a Cartan decomposition, $a\subset p$ a maximal abelian subspace, and we will suppose that $\dim a=\text{rank } V=1$. Let $G=KAN$ denote the related Iwasawa decomposition with components $g=k(g)\exp H(g)n(g)$ and write g_λ for the standard root subspaces in $g^{\sim}$ (here we have positive roots α and possibly 2α). Set $\rho=(1/2)\sum m_\lambda \lambda$ for $\lambda>0$ where $m_\lambda=\dim g_\lambda$ and pick an element $H_0\in a$ with $\alpha(H_0)=1$ while setting $a_t=\exp tH_0$; for $\mu\in \boldsymbol{R}\approx a^*$ we put $\mu(tH_0)=\mu t$ and then $\rho=1/2m_\alpha+m_{2\alpha}$. We identify $(0,\infty)$ with a Weyl chamber $a_+\subset a$. Let M (resp. M') be the centralizer (resp. normalizer) of $A=\exp a$ in K so that the Weyl group (of order $w=2$) is $W=M'/M$ and the boundary of V is $B=K/M$.

Given now $v=gK\in V$ and $b=kM\in B$ one writes $A(v,b)=-H(g^{-1}k)$ and the Fourier transform of $f\in L^2(V)$ is defined by

(1.1) $$\tilde{f}(\mu,b)=\int_V f(v)e^{(i\mu+\rho)A(v,b)}dv$$

for $\mu\in a^*$ and $b\in B$. The inversion formula is

(1.2) $$f(v)=\frac{1}{w}\int_{a^*\times B}\tilde{f}(\mu,b)e^{(-i\mu+\rho)A(v,b)}\,|c(\mu)|^{-2}d\mu db$$

where $c(\mu)$ is the standard Harish-Chandra function (and $w=2$). Now $a^*/W\approx a_+^*$ and one can write

(1.3) $$L^2(V)=\int_{a^*/W}\mathcal{H}_\mu\,|c(\mu)|^{-2}d\mu$$

(1.4) $$\mathcal{H}_\mu=\left\{\phi_\mu(v)=\int_B e^{(-i\mu+\rho)A(v,b)}\varphi(b)db\right\}$$

for $\varphi\in L^2(B)$. The quasiregular representation of G on $L^2(V)$, defined