144. Notes on the Existence of Certain Slit Mappings

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The aim of this article is to give a new type of conformal mappings of plane regions bounded by finitely many analytic Jordan curves. This is achieved by making use of a generalized Riemann-Roch theorem shown in [8]. Also we shall mention about some immediate generalizations.

As is well-known, every plane region is conformally equivalent to a parallel slit region. This theorem was carried over the case of Riemann surfaces with positive finite genus by Kusunoki [3]. Other types of canonical regions can be found in [1], [4]–[6] and in Koebe's classical works (see e.g. [2]). The image region with which we shall deal now is of a different sort from those; it is a finite sheeted covering surface of the extended plane whose boundary consists of slits lying over a fixed straight line.

1. Let R be an arbitrary open Riemann surface of genus $g\,(\leqq+\infty)$ and ∂R its Kerékjártó-Stoïlow ideal boundary. Denote by P a fixed regular partition of ∂R such that $P:\partial R=\alpha\cup\beta\cup\gamma$, where $\phi\subseteqq\alpha\subsetneqq\partial R$. We denote by Q the canonical partition of ∂R (see [1]). Let Λ_0 and Λ_0' be two behavior spaces on R which are dual to each other with respect to $\boldsymbol{R}$ (cf. [7]). Suppose that a $(P)\Lambda_0$-divisor $V_P=V(P,\Lambda_0;\beta,m)$ and a $(Q)\Lambda_0'$-divisor $V_Q=V(Q,\Lambda_0';\gamma,n)$ are given. Consider the ordered pair $\Delta=(V_P,V_Q)$ and set $1/\Delta=\Delta^{-1}=(V_Q,V_P)$. The difference $n-m$ of dimensions is called the index of Δ and is denoted by ind Δ. This definition is different from the preceding one ([8], p.15). Because of this, in the present case we may not distinguish two functions with a constant difference. We set $\mathcal{S}(1/\Delta)=\{f\,|\,$(i) f is a single-valued analytic function on R, (ii) df is a multiple of V_Q, (iii) $\mathfrak{Re}\mathfrak{S}_\beta\, f\tau=0$ for every $\tau\in V_P.\}$ and $\mathcal{D}(\Delta)=\{\omega\,|\,\omega$ is a regular analytic differential on R which is a multiple of V_P and satisfies $\mathfrak{Re}\mathfrak{S}_\gamma\, s\omega=0$ for every $ds\in V_Q.\}$. (As for the definitions of $\mathfrak{Re}\mathfrak{S}_\beta\, f\tau$ etc., see [8].)

Now our Riemann-Roch theorem reads:

Theorem 1 ([8]). *For surfaces of finite genus g,*

$$\dim\mathcal{S}(1/\Delta)-\dim\mathcal{D}(\Delta)=\text{ind}\,\Delta-2g+2.$$

One can find a more general form of the Riemann-Roch theorem in [8].

2. In this section we shall show the following theorem as an ap-