175. *Global Analytic-Hypoellipticity of the $\bar{\partial}$-Neumann Problem*

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(Comm. by Kunihiko KODAIRA, M. J. A., Dec. 12, 1975)

1. Statement of Theorem. Let $M \subset C^n$ be a domain with compact closure $\bar{M}$ and (real)-analytic boundary bM. We denote by r the distance function to bM measured as positive outside and negative inside M. We define Ω'_ρ as the tubular neighborhood of bM in C^n with small width ρ, and set $\Omega_\rho = \bar{M} \cap \Omega'_\rho$. By T_t we denote the subbundle of the complexified tangent bundle CT over Ω'_ρ of all vectors X with $\langle dr, X\rangle = 0$, where $\langle\ ,\ \rangle$ is the duality between covectors and vectors. Splitting CT as $CT = T^{1,0} \oplus T^{0,1}$ with the subbundle $T^{1,0}$ of vectors of type $(1, 0)$ and its complex conjugate $T^{0,1}$, we set $T_t^{1,0} = T^{1,0} \cap T_t$ and $T_t^{0,1} = \overline{T_t^{1,0}}$. Then the *Levi form* at $\mathrm{P} \in \Omega'_\rho$ is defined on the fibre $(T_t^{1,0})_\mathrm{P}$ of $T_t^{1,0}$ at P by

$$(T_t^{1,0})_\mathrm{P} \times (T_t^{1,0})_\mathrm{P} \ni (X_1, X_2) \mapsto \langle \partial\bar{\partial} r, X_1 \wedge \bar{X}_2\rangle.$$

Denote by $\mathcal{A}^{p,q}$ the space of forms of type (p, q) on $\bar{M}$ which have C^∞ extensions to C^n, and define the L^2-inner product by

$$(\varphi, \psi) = \int_M \langle \varphi, \psi\rangle dV, \qquad \varphi, \psi \in \mathcal{A}^{p,q},$$

with the pointwise inner product $\langle\ ,\ \rangle$ and the volume form dV on M. For the Cauchy-Riemann operator $\bar{\partial}: \mathcal{A}^{p,q-1} \to \mathcal{A}^{p,q}$ and its formal adjoint $\vartheta: \mathcal{A}^{p,q} \to \mathcal{A}^{p,q-1}$, integration by parts gives us

$$(\vartheta\varphi, \psi) = (\varphi, \bar{\partial}\psi) + \int_{bM} \langle \sigma(\vartheta, dr)\varphi, \psi\rangle dS,$$

where $\sigma(\cdot, dr)$ denotes the principal symbol of $\cdot$ at dr, and dS the volume form on bM. We set $\mathcal{D}^{p,q} = \{\varphi \in \mathcal{A}^{p,q}; \sigma(\vartheta, dr)\varphi = 0 \text{ on } bM\}$, and define a quadratic form on $\mathcal{D}^{p,q}$ by

$$Q(\varphi, \psi) = (\bar{\partial}\varphi, \bar{\partial}\psi) + (\vartheta\varphi, \vartheta\psi) + (\varphi, \psi), \qquad \varphi, \psi \in \mathcal{D}^{p,q}.$$

Consider the following variational problem (cf. [1], [3]): Given $\lambda \in C$ and $\alpha \in \mathcal{A}^{p,q}$ with $q > 0$, find $\varphi \in \mathcal{D}^{p,q}$ such that

(1) $$Q(\varphi, \psi) + (\lambda\varphi, \psi) = (\alpha, \psi) \qquad \text{for all } \psi \in \mathcal{D}^{p,q}.$$

Now we have

Theorem. *If the Levi form is non-degenerate and does not have exactly q negative eigenvalues in Ω'_ρ, then every solution φ of the equation* (1) *is analytic in Ω_ρ whenever α is analytic there.*

We remark that this Theorem can easily be generalized to the case of domains M in complex manifolds with analytic hermitian metric.