167. *A Remark on the Sobolev Inequality for Riemannian Submanifolds*

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Recently, D. Hoffman and J. Spruck proved a Sobolev inequality in [2] as follows:

Let $M\to\bar{M}$ be an isometric immersion of Riemannian manifolds of dimension m and n, respectively. Using the following quantities:

$\bar{K}_\pi$ = sectional curvature for plane section π in $\bar{M}$,

H = mean curvature vector field of the immersion,

$\bar{R}(M)$ = minimum distance for the cut locus in $\bar{M}$ for all points in M,

ω_m = volume of the unit ball in R^m

and

b = a positive real number

and assuming $\bar{K}_\pi < b^2$, then for any non-negative C^1 function h on M with compact support and $h|\partial M\equiv 0$ we have

$$\left(\int_M h^{m/(m-1)}dV_M\right)^{(m-1)/m}\leqq c(m)\int_M [|\nabla h|+h|H|]dV_M, \tag{1}$$

provided

$$b\left\{\frac{1}{(1-\alpha)\omega_m}\operatorname{Vol}(\operatorname{supp} h)\right\}^{1/m}\leqq 1 \tag{2}$$

and

$$\rho_*:=\frac{1}{b}\sin^{-1}\left[b\left\{\frac{1}{(1-\alpha)\omega_m}\operatorname{Vol}(\operatorname{supp} h)\right\}^{1/m}\right]\leqq\frac{1}{2}\bar{R}(M), \tag{3}$$

where α is a free parameter, $0<\alpha<1$, and

$$c(m)=c(m,\alpha):=\frac{\pi}{2}\cdot\frac{2^{m-2}}{\alpha}\cdot\frac{m}{m-1}\cdot\left\{\frac{1}{(1-\alpha)\omega_m}\right\}^{1/m}. \tag{4}$$

This inequality is very important from the geometric point of view, since this type of inequalities will have a number of interesting applications in differential geometry. In this short paper, we will show that $c(m)$ in (1) must be revised by a more sharper constant, for example

$$c'(m)=c'(m,\alpha,t)$$
$$:=\frac{\pi}{2}\cdot\frac{(m-\alpha)t^{m-1}-(1-\alpha)}{(m-1)\alpha}\cdot\frac{m}{m-1}\cdot\left\{\frac{1}{(1-\alpha)\omega_m}\right\}^{1/m}, \tag{4'}$$

provided (2) and

$$t\rho_*\leqq\bar{R}(M), \tag{3'}$$

where $0<\alpha<1$ and $2\leqq t$.