166. On Closed Countably-Compactifications and Quasi-Perfect Mappings

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Throughout this paper, by a space we shall mean a completely regular T_1-space. According to Morita [14], [15], a space S is a *countably-compactification* ($=c-cf$) of a given space X if

a) S is countably compact ($=cc$) and contains X as a dense subset, and

b) every cc closed subset of X is closed also in S. In case X admits a $c-cf$, X is said to be *countably-compactifiable.* Since X is countably-compactifiable if and only if X has a $c-cf$ S with $X\subset S\subset \beta X$ ([14], Proposition 3.4), in the sequel we will consider only a $c-cf$ S of X as a subspace S of βX with the exception of § 3. Interesting results concerning countably-compactifiability have been obtained by Morita. For example, an M-space X is countably-compactifiable if and only if X is homeomorphic to a closed subset of a product space of a countably compact space and a metric space [14], [15]. In [10] we introduced a notion of closed $c-cf$ and investigated some properties and characterizations of spaces with the closed $c-cf$. Let S be a $c-cf$ of X and $X^*=\beta X-X$ and $S^*=S\cap X^*$. S^* is called *the* X^**-section of* S. In case S^* is closed in X^*, we say that S is the *closed* $c-cf$ *of* X. In Theorem 3.5 [10] it is proved that if X admits a closed $c-cf$, then it is uniquely determined.

Concerning relations between countably-compactifiability of given spaces and maps, it is natural to ask whether countably-compactifiability of X (resp. Y) implies one of Y (resp. X) where Y is a quasi-perfect image of X. For this problem, the following results have been obtained.

Theorem A (Morita [14], Proposition 4.2). *Let f be a perfect map from X onto Y. If Y is countably-compactifiable, then so is X.*

Theorem B (Hoshina [2]). *Let f be a quasi-perfect map from X onto Y and X admits a $c-cf$. Then we have*

1) *if either Y is normal or an M-space, then Y admits a $c-cf$.*

2) *if f is open, then Y admits a $c-cf$.*

Theorem A implies that if f is a perfect map from X onto Y with a $c-cf$ T, then $S=(\beta f)^{-1}T=X\cup S^*$ is a $c-cf$ of X and $f_S=\beta f|S$ is obviously a perfect map from S onto T where $S^*=(\beta f)^{-1}T^*$ and βf is the Stone extension of f. But as shown by Example 3.1, S is not