5. On the Structure of Infinite Transitive Primitive Lie Algebras

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Introduction. In the present paper, we will study the relation between infinite transitive primitive Lie algebra sheaves and their corresponding global Lie algebras. Our proof is done by using the classification theorem of infinite primitive pseudogroups [3]. In the last section, we will calculate the cohomology groups of certain ideals of the Hamiltonian Lie algebra with coefficients in the adjoint representation.

All the results we get in this paper are the extension of the theorems proved by A. Avez, A. Diaz-Miranda and A. Lichnerowicz [1].

1. Preliminaries. Let M be a connected smooth manifold and $\mathfrak{X}(M)$ the Lie algebra of all global smooth vector fields on M. Our main objects are some Lie subalgebras of $\mathfrak{X}(M)$.

It is well known that there are six classes of infinite transitive primitive pseudogroups in the complex analytic case [3]. Now we describe global smooth Lie algebras corresponding to them.

(1) the Lie algebra of all vector fields (i.e. $\mathfrak{X}(M)$),

(2) the Lie algebra of vector fields of divergence zero,

(3) the Lie algebra of vector fields of constant divergence,

(4) the Lie algebra of vector fields preserving a Hamiltonian structure (the Hamiltonian Lie algebra),

(5) the Lie algebra of vector fields preserving a Hamiltonian structure up to constant factors,

(6) the Lie algebra of vector fields preserving a contact structure (the contact Lie algebra).

Let $L(M)$ be one of the global Lie algebras listed above. We denote by $\mathcal{L}$ the Lie algebra sheaf [5] corresponding to it. Let $\mathcal{L}_p$ be a stalk at $p \in M$. Each $\mathcal{L}_p$ has an infinite dimensional Lie algebra structure. We define the linear mapping φ_p of $L(M)$ to $\mathcal{L}_p$ as follows. For any $X \in L(M)$, $\varphi_p(X)$ denotes a germ of X at p. Then φ_p is clearly a Lie homomorphism. It is known that each φ_p is surjective. See for example [4].

The formal algebra L of $\mathcal{L}$ is the Lie algebra consisting of formal Taylor expansions of vector fields belonging to $\mathcal{L}$. For the precise definition, see [5]. Then we have a canonical Lie homomorphism