1. The Exact Functor Theorem for BP_*/I_n-Theory

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(Comm. by Kenjiro SHODA, M. J. A., Jan. 12, 1976)

§ 1. Let p be a prime number and $BP_*(-)$ be the Brown-Peterson homology theory with the coefficient $BP_*\simeq Z_{(p)}[v_1, \cdots]$. Landweber proved the following theorem [4].

Exact functor theorem. *Let G be a BP_*-module and $I_n=(p, v_1, \cdots, v_n)$ be the ideal of BP_* generated by $p, v_1, \cdots, v_n$. Then if the homomorphism*

$$v_{n+1}: G/I_nG\to G/I_nG, \qquad v_{n+1}(g)=v_{n+1}\cdot g,$$

is monic for each $n\geqq -1$, then $BP_*(-)\underset{BP_*}{\otimes} G$ is a homology theory.

On the other hand Sullivan-Baas constructed bordism theories with singularities (Math. Scan. **33**, 1973). Analogously we can define the homology theory $BP(I_n)_*(-)$ with the coefficient $BP_*/I_n\simeq Z_p[v_{n+1}, \cdots]$ [2], [8]. In this paper we shall prove the exact functor theorem for $BP(I_n)_*$-theory.

Theorem. *Let G be a BP_*/I_n-module. If the homomorphism*

$$v_{m+1}: G/I_mG\to G/I_mG$$

is monic for each $m\geqq n$, then $BP(I_n)_*(-)\underset{BP_*/I_n}{\otimes} G$ is a homology theory.

Remark. We always consider *reduced* homology theories in the category of *finite CW*-complexes.

I would like to take this opportunity to thank Professors D. C. Johnson, P. Landweber, S. Sasao and Z. Yosimura for their helpful suggestions in preparing this paper.

§ 2. First, we take argument in the *cohomology theory* $BP(I_n)^*(-)$ which is the *Spanier-Whitehead dual* to $BP(I_n)_*(-)$.

Lemma 1. *Let M be a finitely generated BP^*/I_n- and $BP(I_n)^*(BP(I_n))$-module. Then there exists a BP^*/I_n-filtration such that*

(1) $$M=M_0\supset M_1\supset\cdots\supset M_k=\{0\}$$

(2) $$M_s/M_{s+1}\simeq BP^*/J_s \qquad for\ 0\leqq s<k$$

where J_s is an (invariant) ideal of BP^* satisfying $\theta(J_s)\subset J_s$ for any operation $\theta\in BP^*(BP)$.

Proof. For each $\theta\in BP^*(BP)$, let $\bar\theta_n$ be the set of $\theta_n \in BP(I_n)^*(BP(I_n))$ which commute the following diagram.

$$\begin{array}{ccc} BP & \xrightarrow{i} & BP(I_n) \\ \downarrow{\scriptstyle\theta} & & \downarrow{\scriptstyle\theta_n} \\ S^mBP & \xrightarrow{i} & S^mBP(I_n) \end{array}$$