15. A Note on n-movability and S^k-movability*)

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The following problem was raised by K. Borsuk in [4]. "Let S^k denote the k-dimensional sphere. Does there exist a compactum X which is S^k-movable for $k=1,2,\cdots n$, but is not n-movable?" In this paper, we will construct such a continuum X for the case of $n\geqq 2$.

The concepts of n-movability and A-movability were originally given by K. Borsuk in [3] and [4], and they are equivalent to the following definitions. Let X be a compactum and $X=\{X_n, p_{nn'}, N\}$ be an ANR-sequence associated with X, where each X_n, $n\in N$, is a regular ANR-space (see [6]).

Definition 1 (Y. Kodama and T. Watanabe [6]). A compactum X is said to be *k-movable* if for each $n\in N$ there is $n'\in N$, $n'\geqq n$, such that for each $n''\in N$, $n''\geqq n$, and for each compact set $K\subset X_n$, with $\dim K\leqq k$, there is a map $f_{n''}: K\to X_{n''}$ satisfying the homotopy relation: $p_{nn''}f_{n''}\simeq p_{nn'}|_K: K\to X_n$.

Definition 2. Let A be a compactum. A compactum X is said to be *A-movable* if for each $n\in N$ there is $n'\in N$, $n'\geqq n$, such that for each $n''\in N$, $n''\geqq n$, and for each map $f: A\to X_{n'}$, there is a map $f_{n''}: A\to X_{n''}$ satisfying the homotopy relation: $p_{nn''}f_{n''}\simeq p_{nn'}f: A\to X_n$.

The equivalence of the concept of A-movability in Definition 2 and the original one can be shown by the same way as in the proof of Theorem 3 of [5].

Our example is homeomorphic to the continuum constructed by K. Borsuk [2]. For completeness we give its construction. Consider the following compact subsets of an Euclidean 3-space R^3:

$$\begin{aligned}
A_1&=\{(x,y,z)\,|\,x^2+y^2+z^2=5, |x|\leqq 2, |z|\leqq 2,\}\\
&\quad\cup\{(x,y,z)\,|\,x^2+y^2=1, |z|\leqq 2\}\\
B_0&=\{(x,y,z)\,|\,x^2+y^2+z^2=5, |x|\leqq -2\}\\
B_1&=\{(x,y,z)\,|\,x^2+y^2+z^2=5, x\geqq 2\}\\
A_n&=\{(x,y,z)\,|\,(x-4n+4,y,z)\in A_1\}, \qquad n=2,3,\cdots\\
B_n&=\{(x,y,z)\,|\,(x-4n+4,y,z)\in B_1\}, \qquad n=2,3,\cdots.
\end{aligned}$$

Put $X_n=B_0\cup A_1\cup A_2\cup\cdots\cup A_n\cup B_n$, for each $n\in N$. For $n,n'\in N$, $n'>n$, define a map $p_{nn'}: X_{n'}\to X_n$ by

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