14. *Propagation of Singularities of Solutions for Pseudodifferential Operators with Multiple Characteristics and their Local Solvability*

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In this note we consider propagation of singularities of solutions for pseudodifferential operators with multiple real characteristics and local solvability. It has become clearer from recent investigations that localization in (x, ξ)-space (micro-localization) is useful in order to study regularity and solvability. Our results stated in this note are obtained by micro-local considerations. Details will be published elsewhere.

Let Ω be a domain in $\boldsymbol{R}^n$. (x, ξ) denotes a point in $\Omega\times(\boldsymbol{R}^n-\{0\})$ and $D=\dfrac{1}{\sqrt{-1}}\dfrac{\partial}{\partial x}$. $P(x, D)\in L^m(\Omega)$ means that $P(x, D)$ is a classical pseudodifferential operators of order m. Let Γ be an open conic subset of $\Omega\times(\boldsymbol{R}^n-\{0\})$. We set $L^m(\Omega, \Gamma)=\{P(x, D)\in L^m(\Omega)\,;\ \operatorname{supp} p(x, \xi)\subset\Gamma\}$, where $p(x, \xi)$ is the symbol of $P(x, D)$.

Definition. An operator $P(x, D)\in L^m(\Omega)$ has constant multiple real characteristics, if the principal symbol $p_m(x, \xi)$ is decomposed into

$$p_m=(p^1)^{m_1}(p^2)^{m_2}\cdots(p^s)^{m_s}, \tag{1}$$

where p^i $(i=1, 2, \cdots s)$ has the following properties:

Each p^i is real valued, positively homogenous in ξ and $\operatorname{grad}_\xi p^i\neq 0$ on the characteristic manifold $\Sigma_{p^i}=\{(x, \xi)\,;\ p^i(x, \xi)=0\}$ and if $j\neq k$, $\Sigma_{p^j}\cap\Sigma_{p^k}=\phi$.

Hereafter we assume that $P(x, D)\in L^m(\Omega)$ is properly supported and has constant multiple real characteristics. So in a conic neighbourhood Γ of $(x_0, \xi_0)\in\Sigma_p=\{(x, \xi)\,;\ p_m(x, \xi)=0\}$ we can factorize the principal symbol as follows:

$$p_m(x, \xi)=a(x, \xi)q(x, \xi)^k, \tag{2}$$

where $q(x, \xi)$ is real valued, positively homogenous of order 1 in ξ, and $q(x_0, \xi_0)=0$, $\operatorname{grad}_\xi q(x_0, \xi_0)\neq 0$ and $a(x, \xi)$ is real valued, positively homogenous of order $m-k$ in ξ and $a(x, \xi)\neq 0$ on Γ.

In order to micro-localize our study, we introduce a function $\varphi(x, \xi)\geqq 0$ positively homogenous of order 0 in ξ such that $\varphi\equiv 1$ in a conic neighbourhood $\Gamma_1(\Gamma_1\subset\Gamma)$ of (x_0, ξ_0) and $\varphi\equiv 0$ outside of Γ.

As for micro-local regularity, we have

Theorem 1. *Suppose that there are* $B^i_j(x, D)\in L^j(\Omega)$ *with the*