30. *A Note on Explosion of Branching Markov Processes with Extinction*

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1. Preliminary. We discuss the explosion problem of branching Markov process under extinction effect. Such a problem was not considered in [3] and [4], since the existence of extinction brings some difficulty on the probabilistic consideration.[1] The difficulty will be removed through the auxiliary procedure which will be presented below.

Let S be a locally compact Hausdorff space with the second countability. Let $\boldsymbol{S}$ be the topological sum of the symmetric product spaces $S^{(n)}$, $n=0,1,\cdots,\infty$, with $S^{(0)}=\{\partial\}$ and $S^{(\infty)}=\{\Delta\}$. Let $\boldsymbol{X}=(\Omega,\boldsymbol{X}_t,\boldsymbol{P}_x)$ be a branching Markov process on the state space $\boldsymbol{S}$ in the sense of [1]. For $\boldsymbol{X}$ define the extinction time by $e_\partial=\inf\{t;\boldsymbol{X}_t=\partial\}$ and the explosion time by $e_\Delta=\inf\{t;\boldsymbol{X}_t=\Delta\}$.[2] Let $\{\boldsymbol{T}_t\}_{t\geqslant 0}$ be the semi-group of $\boldsymbol{X}$ acting on $C_0(\boldsymbol{S})$.[3] Set $q(x)=\lim_{t\to\infty}\boldsymbol{T}_t\hat{0}(x)=\boldsymbol{P}_x(e_\partial<\infty)$ for $x\in S$, where for each function f on S a function $\hat{f}$ on $\boldsymbol{S}$ is defined as follows; $\hat{f}(\partial)=1$, $\hat{f}(\Delta)=0$ and $\hat{f}(\boldsymbol{x})=f(x_1)\cdots f(x_n)$ if $\boldsymbol{x}=[x_1,\cdots,x_n]\in S^{(n)}$, $n=1,2,\cdots$. Throughout this article we assume

(*Asm.*) *$q(x)$ is a continuous function on S such that $0\leqslant q(x)<1$, $x\in S$.*

Let us define the family of operators $\{\tilde{\boldsymbol{T}}_t\}_{t\geqslant 0}$ for $\hat{f}\in C_0(\boldsymbol{S})$ with a continuous function f on S such that $0\leqslant f(x)<1$ for $x\in S$.

$$(1)\qquad \tilde{\boldsymbol{T}}_t\hat{f}(x)=\frac{1}{1-q(x)}\{\boldsymbol{T}_t(\widehat{q+(1-q)f})(x)-q(x)\},\qquad x\in S.$$

Following [1] $\{\tilde{\boldsymbol{T}}_t\}_{t\geqslant 0}$ is uniquely extended to a branching semi-group acting on $C_0(\boldsymbol{S})$, and we also denote the extension by $\{\tilde{\boldsymbol{T}}_t\}_{t\geqslant 0}$. $\{\tilde{\boldsymbol{T}}_t\}_{t\geqslant 0}$ determines a branching Markov process $\tilde{\boldsymbol{X}}$ on $\boldsymbol{S}$ (cf. [1]). We call the process $\tilde{\boldsymbol{X}}$ the associated (branching Markov) process to $\boldsymbol{X}$.

2. Results and the proof.

Lemma 1. *Let $\tilde{\boldsymbol{X}}$ be the associated process to $\boldsymbol{X}$, then*

(i) *$\boldsymbol{X}$ is explosive if and only if $\tilde{\boldsymbol{X}}$ is explosive.*

(ii) *If $\tilde{\boldsymbol{X}}$ is explosive with probability one, then*

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1) For the terminologies used in our note, refer [3] and [4].

2) We define $\inf\{\emptyset\}=\infty$.

3) $C_0(\boldsymbol{S})=\{f;$ continuous function on $\boldsymbol{S}$ which vanishes at the infinities of $\boldsymbol{S}\}$, where the infinities consist of Δ and the infinity of the one point compactification of $S^{(n)}$, $n=1,2,\cdots$.