# 28. On a Nonlinear Noncontractive Semigroup

By Naoki YAMADA
Department of Mathematics, Kobe University

(Comm. by Kôsaku YOSIDA, M. J. A., March 12, 1976)

**1. Introduction and Theorem.** Let $X$ be a Banach space with norm $\|\cdot\|$. We consider an operator $A: D(A)\subset X\to X$ such that i) $D(A)\ni 0$, $A0=0$ ii) $R(I+\lambda A)=X$ *for all* $\lambda>0$ iii) *there exists a constant* $M>0$ *such that for all* $\lambda>0$ *and* $x, y\in X$,

$$\|(I+\lambda A)^{-1}x-(I+\lambda A)^{-1}y\|\le M\|x-y\|.$$

Let $J_\lambda=(I+\lambda A)^{-1}$ be Fréchet differentiable at every $x\in X$. Then $F(\lambda)=J'_\lambda[x+\lambda Ax]\in B(X,X)(x\in D(A))$ satisfies the first resolvent equation; $\lambda F(\lambda)-\mu F(\mu)=(\lambda-\mu)F(\mu)F(\lambda)$ (see [3] or [4]). Hence it follows that there exists a linear operator $A'[x]: D(A'[x])\to X$ such that $F(\lambda)=(I+\lambda A'[x])^{-1}$. Such an operator $A$ is said to be $R$-defferentiable and $A'[x]$ the $R$-derivative of $A$ at $x\in D(A)$.

The notion of $R$-differentiable operators was introduced by M. Iannelli to construct nonlinear noncontractive semigroups. In this note, we shall consider an $R$-differentiable opetator $A$ such that $A'[x]$ satisfies a hyperbolic-type condition. We shall show that the infinitesimal generator of a semigroup associated with $A$, coincides with $A$ on a subspace of $X$. Only the result and an outline of its proof are presented here and the details will be published elsewhere. Our result is following

**Theorem.** *Let $A$ be an $R$-differentiable operator such that:*

( I ) *$A'[x]$ is a closed linear operator for all $x\in D(A)$,*

( II ) *there exists a Banach space $Y$ which is densely and continuously embedded in $X$,*

($S_1$) *for any finite family $\{x_1,\cdots,x_n\}\subset D(A)$,*

$$\left\|\prod_{i=1}^{n}(I+\lambda A'[x_i])^{-1}\right\|_X\le M,$$

($S_2$) *$(I+\lambda A'[x])^{-1}(Y)\subset Y$ for each $x\in D(A)$, and for $\{x_i\}$ stated in ($S_1$),*

$$\left\|\prod_{i=1}^{n}(I+\lambda A'[x_i])^{-1}\right\|_Y\le K_1,$$

(III) *$Y\subset D(A)$, $Y\subset D(A'[x])$ for each $x\in D(A)$, and*

$$\|A'[x]-A'[y]\|_{Y,X}\le K_2\|x-y\|.$$

*Here $K_i$, $i=1,2$ are constants and $\|\cdot\|_X$, $\|\cdot\|_Y$, $\|\cdot\|_{Y,X}$ denote the norms in $B(X,X)$, $B(Y,Y)$, $B(Y,X)$ respectively.*

*Then there exists a unique semigroup $\{G(t)\}_{t\ge 0}$ such that*

(a) *$G(t)x=\lim_{n\to\infty}(I+(t/n)A)^{-n}x$ for all $t\ge 0$ and $x\in X$,*