26. On the Irreducible Characters of the Finite Unitary Groups

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Let k be a finite field, and k_2 the quadratic extension of k. The purpose of the present paper is to announce a theorem which gives a method to construct the irreducible characters of the finite unitary group $U_n(k_2)$ using those of the finite general linear group $GL_n(k_2)$, at least if the characteristic of k is not 2. As an application, we also obtain a parametrization of the irreducible characters of $U_n(k_2)$ which is dual to a known parametrization of the conjugacy classes. Proofs are omitted and will appear elsewhere.

1. Let $\mathfrak{G}$ be the general linear group $GL_n(K)$ over an algebraically closed field K of positive characteristic p. Let k be a finite subfield of K, and $k_m(\subset K)$ the extension of k of degree $m<\infty$. We denote by τ the Frobenius automorphism of K with respect to k. Then τ acts naturally on $\mathfrak{G}$ as an automorphism. Let σ be the automorphism of $\mathfrak{G}$ defined by

$$x^\sigma=(({}^t x)^\tau)^{-1} \qquad (x\in\mathfrak{G}),$$

where ${}^t x$ is the transposed matrix of $x\in\mathfrak{G}$. For a positive integer m, put

$$\mathfrak{G}_{\sigma^m}=\{x\in G \mid x^{\sigma^m}=x\}.$$

Then we have

$$\mathfrak{G}_{\sigma^m}=\begin{cases} GL_n(k_m) & \text{if } m \text{ is even,} \\ U_n(k_{2m}) & \text{if } m \text{ is odd.}\end{cases}$$

In the following, we fix m and put $G=\mathfrak{G}_{\sigma^m}$ and $G_\sigma=\mathfrak{G}_\sigma=U_n(k_2)$. The restriction of σ to G is an automorphism of the finite group G. In the following, we denote this automorphism also by σ. Let A be the cyclic group of order m generated by the automorphism σ of G. Assume that G and A are embedded in their semi-direct product GA. The following lemma is well known.

Lemma 1. *Let H be a finite group, and A a finite cyclic group generated by an automorphism σ of H. If an irreducible complex character χ of H is fixed by σ (i.e. satisfies $\chi(x^\sigma)=\chi(x)$ for all $x\in H$), then there exists an irreducible character $\tilde{\chi}$ of the semi-direct product HA whose restriction to H equals χ.*

For $x\in G=\mathfrak{G}_{\sigma^m}$, put $N(x)=xx^\sigma x^{\sigma^2}\cdots x^{\sigma^{m-1}}$.