50. **On an Explicit Formula for Class-1 "Whittaker Functions" on GL_n over \( p \)-adic Fields**

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"Whittaker functions" on \( p \)-adic linear groups have been studied by several authors (see e.g. [2] and [3]). In this note, we present an explicit formula for the class-1 "Whittaker functions" on \( GL_n(k) \), where \( k \) is a non archimedean local field.

1. Let \( k \) be a finite extension of the \( p \)-adic field \( \mathbb{Q}_p \) and let \( \mathcal{O} \) be the ring of integers of \( k \). Choose a generator \( \pi \) of the maximal ideal of \( \mathcal{O} \) and denote by \( q \) the cardinality of the residue class field of \( k \). Set \( G=GL_n(k) \) and \( K=GL_n(\mathcal{O}) \). Then \( K \) is a maximal compact open subgroup of \( G \). The invariant measure of \( G \) is normalized so that the total volume of \( K \) is equal to 1. Denote by \( L_0(G,K) \) the space of complex valued compactly-supported bi-\( K \)-invariant functions on \( G \). Then \( L_0(G,K) \) is a commutative subalgebra of the group ring \( L(G) \) of \( G \).

We denote by \( N \) the group of \( n \times n \) upper triangular unipotent matrices with entries in \( k \). Choose a character \( \psi \) of the additive group of \( k \) which is trivial on \( \mathcal{O} \) but not trivial on \( \pi^{-1}(\mathcal{O}) \). Denote by the same letter \( \psi \) the character of \( N \) given by \( \psi(x)=\prod_{i=1}^{n-1} \psi(x_{i,i+1}) \), where \( x_{i,i+1} \) is the \((i,i+1)\) entry of \( x \).

For each algebra homomorphism \( \lambda \) of \( L_0(G,K) \) into \( \mathbb{C} \), it is known that there uniquely exists a function \( W_\lambda(g) \) on \( G \) which satisfies the following conditions (1), (2) and (3).

1. \( W_\lambda(xg)=\psi(x)W_\lambda(g) \) \( (\forall x \in N) \),
2. \( \int_G W_\lambda(gx)\varphi(x)dx=\lambda(\varphi)W_\lambda(g) \) \( (\forall \varphi \in L_0(G,K)) \),
3. \( W_\lambda(1)=1 \).

The function \( W_\lambda \) is said to be the class-1 "Whittaker function" on \( G \) associated with the homomorphism \( \lambda \) of \( L_0(G,K) \) into \( \mathbb{C} \).

For each \( n \)-tuple \( f=(f_1,f_2,\ldots,f_n) \) of integers, we denote by \( \pi^f \) the diagonal matrix whose \( i \)-th diagonal entry is \( \pi^{f_i} \) \( (i=1,\ldots,n) \). Set \( w_\lambda(f)=W_\lambda(\pi^f) \). It is known that \( G=\bigsqcup_{f \in \mathbb{Z}^n} N\pi^f K \) (disjoint union). To evaluate \( W_\lambda \) on \( G \), it is sufficient to know \( w_\lambda(f) \) for all \( f \in \mathbb{Z}^n \), since \( W_\lambda \) is right \( K \)-invariant and satisfies (1). Since the conductor of \( \psi \) is \( \mathcal{O} \), it follows easily from (1) that \( w_\lambda(f) \) is zero unless \( f_1 \geq f_2 \geq \cdots \geq f_n \).

For \( i=1,2,\ldots,n \), let \( \varphi_i \) be the characteristic function of the double...