50. On an Explicit Formula for Class-1 "Whittaker Functions" on GL_n over $\mathfrak{P}$-adic Fields

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0. "Whittaker functions" on $\mathfrak{P}$-adic linear groups have been studied by several authors (see e.g. [2] and [3]). In this note, we present an explicit formula for the class-1 "Whittaker functions" on $GL_n(k)$, where k is a non archimedean local field.

1. Let k be a finite extension of the p-adic fied $\boldsymbol{Q}_p$ and let $\mathcal{O}$ be the ring of integers of k. Choose a generator π of the maximal ideal of $\mathcal{O}$ and denote by q the cardinality of the residue class field of k. Set $G=GL_n(k)$ and $K=GL_n(\mathcal{O})$. Then K is a maximal compact open subgroup of G. The invariant measure of G is normalized so that the total volume of K is equal to 1. Denote by $L_0(G,K)$ the space of complex valued compactly-supported bi-K-invariant functions on G. Then $L_0(G,K)$ is a commutative subalgebra of the group ring $L^1(G)$ of G. We denote by N the group of $n\times n$ upper triangular unipotent matrices with entries in k. Choose a character ψ of the additive group of k which is trivial on $\mathcal{O}$ but not trivial on $\pi^{-1}\mathcal{O}$. Denote by the same letter ψ the character of N given by $\psi(x)=\prod_{i=1}^{n-1}\psi(x_{ii+1})$, where x_{ii+1} is the $(i,i+1)$ entry of x.

For each algebra homomorphism λ of $L_0(G,K)$ into $\boldsymbol{C}$, it is known that there uniquely exists a function $W_\lambda(g)$ on G which satisfies the following conditions (1), (2) and (3).

$$W_\lambda(xg)=\psi(x)W_\lambda(g) \qquad (\forall x\in N), \tag{1}$$

$$\int_G W_\lambda(gx)\varphi(x)dx=\lambda(\varphi)W_\lambda(g) \qquad (\forall \varphi\in L_0(G,K)), \tag{2}$$

$$W_\lambda(1)=1. \tag{3}$$

The function W_λ is said to be the *class*-1 "*Whittaker function*" *on G associated with the homomorphism λ of $L_0(G,K)$ into $\boldsymbol{C}$.*

For each n-tuple $f=(f_1,f_2,\cdots,f_n)$ of integers, we denote by π^f the diagonal matrix whose i-th diagonal entry is π^{f_i} $(i=1,\cdots,n)$. Set $w_\lambda(f)=W_\lambda(\pi^f)$. It is known that $G=\bigcup_{f\in \boldsymbol{Z}^n} N\pi^f K$ (disjoint union). To evaluate W_λ on G, it is sufficient to know $w_\lambda(f)$ for all $f\in\boldsymbol{Z}^n$, since W_λ is right K-invariant and satisfies (1). Since the conductor of ψ is $\mathcal{O}$, it follows easily from (1) that $w_\lambda(f)$ is zero unless $f_1\geq f_2\geq\cdots\geq f_n$.

For $i=1,2,\cdots,n$, let φ_i be the characteristic function of the double