46. *On the Cauchy Problem for Weakly Hyperbolic Systems*

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§1. **Introduction.** In this paper we consider the $\mathcal{E}$-well-posedness for the Cauchy problem of the first order system:

$$
(1.1)\qquad \begin{cases} M[u]=\dfrac{\partial}{\partial t}u-\sum\limits_{j=1}^{l}A_j(x,t)\dfrac{\partial}{\partial x_j}u-B(x,t)u=f(x,t), \\ \qquad\qquad\qquad\qquad (x,t)\in\Omega=R_x^l\times[0,T], \\ u(x,t_0)=u_0(x), \qquad 0\leqq t_0<T, \end{cases}
$$

where $A_j(x,t)$ and $B(x,t)$ are (m,m) matrices whose elements belong to the class $\mathcal{B}(\Omega)$ (in the sense of L. Schwartz [5]).

We suppose that $A(x,t,\xi)=\sum_{j=1}^{l}A_j(x,t)\xi_j$ is not diagonalizable. Such a case has been treated by V. M. Petkov with the method of asymptotic expansions ([6], [7]).

Here we shall approach this problem in a different viewpoint from his and propose a more concrete condition which is necessary and sufficient for the $\mathcal{E}$-well-posedness of (1.1). Our proof is much due to, so-called, the method of energy estimates (see S. Mizohata [2], S. Mizohata and Y. Ohya [3], [4]). The forthcoming paper will give the detailed proofs.

§2. **Levi's condition and an energy estimate.** As indicated in §1, throughout this paper we assume the following:

(2.1) The multiplicities of the characteristic roots are constant and at most double, more precisely,

$$
\det(\tau I-A(x,t;\xi))=\prod_{i=1}^{s}(\tau-\lambda_i(x,t;\xi))^2\qquad\prod_{j=s+1}^{m-s}(\tau-\lambda_j(x,t;\xi)).
$$

(2.2) The roots $\lambda_i(x,t;\xi)$ are real and distinct for $(x,t;\xi)\in\Omega\times(R_\xi^l\setminus\{0\})$, $(i=1,2,\cdots,m-s)$.

(2.3) For $i=1,2,\cdots,s$, $\operatorname{rank}(\lambda_i(x,t;\xi)I-A(x,t;\xi))=m-1$, independently of $(x,t;\xi)$.

Proposition 2.1. *Suppose* (2.1) *and* (2.3), *then there exists a* (m,m) *matrix* $N(x,t;\xi)$ *which satisfies*

(i) $N(x,t;\xi)A(x,t;\xi)=D(x,t;\xi)N(x,t;\xi)$,

where