

## 45. On Quadratic Differentials with Closed Trajectories and their Applications

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In this paper we shall consider on compact Riemann surfaces the class of quadratic differentials with finite norm and closed trajectories and state some theorems, the detailed proofs of which will be given in another paper ([5]) together with related results.

1. Let  $R$  be a compact Riemann surface of genus  $g (> 0)$ , we write

$$\tilde{A}_1 = \tilde{A}_1(R) = \{\theta : \theta \text{ is a holomorphic abelian differential on } R.\}$$

$$\tilde{A}_2 = \tilde{A}_2(R) = \{\theta : \theta \text{ is a holomorphic quadratic differential on } R.\}$$

$$A_2D = A_2D(R) = \left\{ \phi : \phi \text{ is a meromorphic quadratic differential on } R \right.$$

such that  $|||\phi||| = \int_R |\phi| < +\infty. \left. \right\}$

$$CA_2D = CA_2D(R) = \{\phi \in A_2D \text{ with closed trajectories.}\}$$

$$C\tilde{A}_1 = C\tilde{A}_1(R) = \{\theta \in \tilde{A}_1 : \theta^2 \in CA_2D.\}$$

$$C\tilde{A}_2 = C\tilde{A}_2(R) = CA_2D \cap \tilde{A}_2.$$

Then we can prove the following

**Proposition 1** (cf. [1]). *Let  $\gamma$  be an arbitrary 1-cycle on  $R$ . Then the holomorphic reproducing differential  $\theta_\gamma$  for  $\gamma$  belongs to  $C\tilde{A}_1$ .*

And using this proposition, we have

**Proposition 2.** *The set  $C\tilde{A}_1$  is dense in  $\tilde{A}_1$  with respect to the Dirichlet norm.*

Thus making two (or four if necessary) sheeted covering surface  $R'$  of  $R$ , and considering the class of odd holomorphic reproducing differentials on  $R'$ , we can prove the following Strebel's conjecture ([3]).

**Theorem 1.** *The set  $CA_2D$  is dense in  $A_2D$  with respect to the  $|||$ -norm. Moreover if  $\phi \in A_2D$  has poles at  $\{P_i\}_{i=1}^r$ , then  $\phi$  can be approximated by the elements of  $CA_2D$  with poles at  $\{P_i\}_{i=1}^r$ .*

The last assertion follows from the fact that the norm convergence is equivalent to the locally uniform convergence.

2. The set of holomorphic reproducing differentials is dense in  $\tilde{A}_1$ . But an element of  $C\tilde{A}_1$  is not always proportional to some holomorphic reproducing differential in the case that  $g \geq 2$ . This follows from the following