79. On the Singularities of the Riemann Functions of Mixed Problems for the Wave Equation in Plane-Stratified Media. I

By Mutsuhide MATSUMURA
Faculty of Science, Tokyo University of Education

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1. Introduction. The study on the singularities of the fundamental solutions (or Riemann functions) of mixed initial boundary value problems for linear hyperbolic equation with constant coefficients in a quarter space has been developed primarily by Duff [2] and afterward by Deakin [1], Matsumura [6], Wakabayashi [9], Tsuji [8], especially by Wakabayashi [10] and [11]. The purpose of this series of notes is to show that the methods in [6], [8]–[11] are applicable to the study of the singularities of the Riemann functions of mixed initial boundary value problems with a plane interface in a quarter space for the wave equation. This problem was suggested by Wilcox [12].

2. Formulation of the problem and Lopatinski's determinant. $\boldsymbol{R}^n$ denotes the n-dimensional Euclidean space and $\boldsymbol{\Xi}^n$ denotes its real dual space with duality $\langle x, \xi\rangle = x_1\xi_1 + \cdots + x_n\xi_n$. Let us write $x' = (x_1, \cdots, x_{n-1})$, $x'' = (x_2, \cdots, x_n)$ for the coordinate $x = (x_1, \cdots, x_n)$ in $\boldsymbol{R}^n$ and $\xi' = (\xi_1, \cdots, \xi_{n-1})$, $\xi'' = (\xi_2, \cdots, \xi_n)$ for the dual coordinate $\xi = (\xi_1, \cdots, \xi_n)$ in $\boldsymbol{\Xi}^n$. x_1 will play the role of time variable and x'' will play the role of physical space variable. Let h be a given positive number, and set $\Omega_I = \{x'' \in \boldsymbol{R}^{n-1}; 0 < x_n < h\}$ and $\Omega_{II} = \{x'' \in \boldsymbol{R}^{n-1}; x_n > h\}$. We consider two wave operators $P_1(D) = a_1^2\Delta - D_1^2$ and $P_2(D) = a_2^2\Delta - D_1^2$ with wave speeds $a_1 > 0$ and $a_2 > 0$ which govern the wave propagation in Ω_I and Ω_{II}, respectively. Here $D_j = \partial/i\partial x_j$ and $\Delta = D_2^2 + \cdots + D_n^2$. The mixed problem we will study is

(1) $P_1(D)u(x) = f(x),\ x_1 > 0,\ 0 < x_n < h$ (i.e. in $\boldsymbol{R}_+ \times \Omega_I$),

(2) $P_2(D)u(x) = f(x),\ x_1 > 0,\ x_n > h$ (i.e. in $\boldsymbol{R}_+ \times \Omega_{II}$),

(3) $u(0, x'') = g_0(x''),\ (D_1 u)(0, x'') = g_1(x'')$ (initial conditions),

(4) $Q(D)u(x)|_{x_n=0} = k_0(x'),\ x_1 > 0$ (boundary condition)

(5) $B_j(D)u(x)|_{x_n=h-} = C_j(D)u(x)|_{x_n=h+} + k_j(x'),\ x_1 > 0,\ j = 1, 2$ (interface or transmission conditions),

where $Q(D)$, $B_j(D)$ and $C_j(D)$ are partial differential operators with constant coefficients.

Let Γ_ι denote the cone $\{\eta \in \boldsymbol{\Xi}^n; \eta_1 > 0, \eta_1^2 > a_\iota^2|\eta''|^2\}$ and let us denote by $\lambda_\iota^+ = \lambda_\iota^+(\xi' + i\eta')$ and $\lambda_\iota^- = \lambda_\iota^-(\xi' + i\eta')$ the roots with positive and negative