77. *On the System of Pfaffian Equations of Briot-Bouquet Type*

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§1. **Introduction.** In this paper we shall extend some well-known results on the system of ordinary differential equations of Briot-Bouquet type to the system of Pfaffian equations. By a system of Pfaffian equations of Briot-Bouquet type we mean a completely integrable system of Pfaffian equations

$$du_i=\sum_{k=1}^{n}\frac{f^{ik}(u_1,\cdots,u_m,x_1,\cdots,x_n)}{x_k}dx_k,\qquad i=1,\cdots,m,$$

or

$$(1)\qquad x_k\frac{\partial u_i}{\partial x_k}=f^{ik}(u,x),\qquad i=1,\cdots,m;\ k=1,\cdots,n,$$

where the f^{ik} are functions holomorphic at the origin $u_1=\cdots=u_m=x_1=\cdots=x_n=0$ and vanishing there. By the use of the usual multi-index notation: $\alpha=(\alpha_1,\cdots,\alpha_m)$, $\beta=(\beta_1,\cdots,\beta_n)$, the Taylor expansions of the f^{ik} are expressible as

$$f^{ik}(u,x)=\sum_{\mu=1}^{m}a_{i\mu}^{k}u_\mu+\sum_{\nu=1}^{n}a_\nu^{ik}x_\nu+\sum_{|\alpha|+|\beta|\geq 2}a_{\alpha\beta}^{ik}u^\alpha x^\beta.$$

By denoting A_k the matrix formed by the coefficients of $u_1,\cdots,u_m$ in the developments of $f^{1k},\cdots,f^{mk}$, let $\lambda_1^k,\cdots,\lambda_m^k$ be the eigenvalues of A_k.

The complete integrability condition for (1) can be written as follows:

$$(2)\qquad \sum_{\mu=1}^{m}\frac{\partial f^{il}}{\partial u_\mu}f^{\mu k}+x_k\frac{\partial f^{il}}{\partial x_k}=\sum_{\mu=1}^{m}\frac{\partial f^{ik}}{\partial u_\mu}f^{\mu l}+x_l\frac{\partial f^{ik}}{\partial x_l}.$$

§2. **Formal integration.**

Theorem 2.1. *Suppose that*

(i) *All the A_k, $k=1,\cdots,n$, are similar to diagonal matrices;*

(ii) *For any system of non-negative integers $(\alpha_1,\cdots,\alpha_m,B)$, there exists an index K, $1\leq K\leq n$, such that*

$$\lambda_i^K\neq\sum_{\mu=1}^{m}\alpha_\mu\lambda_\mu^K+B,\qquad i=1,\cdots,m.$$

Then there exists a formal transformation of the form

$$(3)\qquad u_i=\sum_{\mu=1}^{m}p_{i\mu}v_\mu+\sum_{\nu=1}^{n}p_\nu^i x_\nu+\sum_{|\alpha|+|\beta|\geq 2}p_{\alpha\beta}^i v^\alpha x^\beta,$$

which transforms the system (1) *into the system*