# 76. *On Some Additive Divisor Problems. II*

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§ 1. In our previous paper [4] we have given a very simple proof of the asymptotic formula (as $N\to\infty$)

$$\begin{aligned}\mathrm{D}_k(N;a)&=\sum_{n\leqq N}\mathrm{d}_2(n+a)\mathrm{d}_k(n)\\&=S_k(a)N(\log N)^k+O(N(\log N)^{k-1}\operatorname{loglog}N),\end{aligned}$$

where $a$ is a fixed integer, $\mathrm{d}_k(n)$ the coefficient of $\zeta(s)^k$, $k\geqq 3$ arbitrary. The problem for general $k$ has been firstly treated by Linnik in his book [3]. There it is indicated also that his method enables us to deduce even an expansion with an error-term $O(N(\log N)^\varepsilon)$, $\varepsilon>0$ being arbitrarily small (see also Bredikhin [1]). But it seems that neither Linnik nor Bredikhin have been able to eliminate this error-term.

Now the purpose of this note is to announce

**Theorem.** *We have the asymptotic expansion, for arbitrary* $k$,

$$\mathrm{D}_k(N;a)=N\sum_{j=0}^{k}f_k^{(j)}(a)(\log N)^j+O(N(\log N)^{-1+\varepsilon}).$$

The coefficients can be calculated, but at the cost of big labour. The result should be compared with Estermann's asymptotic expansion for the case of $k=2$ ([2]).

§ 2. We indicate very briefly the main steps of our proof, whose detailed exposition will appear elsewhere.

Now by an obvious reason it is sufficient to consider the case of $a=1$. And we prove that, denoting by $(P)$ the set of primes in the interval $(N^{3/4}, N(\log N)^{-A})$ with sufficiently large $A$, we have

$$(*)\qquad \mathrm{D}_k(N;1)-\mathrm{D}_k(N;p)=O(N(\log N)^{-1+\varepsilon}),$$

uniformly for all $p\in(P)$. To do this we divide $\mathrm{D}_k(N;a)$ into two parts. Let $z_1=\exp((\log N)^{\varepsilon_1})$, $\varepsilon_1=\varepsilon/(3k+1)$, $z_2=\exp((\log N)(\operatorname{loglog}N)^{-2})$, and further let (I), (II) be two sets of integers $\leqq N$ such that $n\in$ (I) has no prime factors in the interval $(z_1,z_2)$ and (II) is the complementary set of (I). And we put, $a$ being 1 or $p\in(P)$,

$$\mathrm{D}_k(N;a)=\sum_{n\in(\mathrm{I})}+\sum_{n\in(\mathrm{II})}=\mathrm{D}_k^{(1)}(N;a)+\mathrm{D}_k^{(2)}(N;a).$$

By a direct application of the dispersion method [3] we can show that

**Lemma 1.** *We have, uniformly for all* $p\in(P)$,

$$\mathrm{D}_k^{(2)}(N;1)-\mathrm{D}_k^{(2)}(N;p)=O(N(\log N)^{-2}).$$

§ 3. As for $\mathrm{D}_k^{(1)}(N;a)$ we first define another two sets of integers