74. *An Induction Principle for the Generalization of Bombieri's Prime Number Theorem*

By Yoichi MOTOHASHI
Department of Mathematics, College of Science and Technology, Nihon University, Tokyo

(Communicated by Kenjiro SHODA, M. J. A., June 8, 1976)

1. Recently several authors have considered generalizations into various directions of Bombieri's prime number theorem [2]. Here we give an induction principle through which most of former results follow in improved forms and also with which we can expand considerably the domain of the equi-distributed sequences (for this terminology see [1]).

Let f be a complex valued arithmetic function, and let introduce the following properties. $(\mathcal{A})$: $f(n)=O(\tau(n)^C)$, where $\tau(n)$ is the divisor function. $(\mathcal{B})$: If the conductor of a non-principal character χ is $O((\log x)^D)$, then we have $\sum_{n\leqq x} f(n)\chi(n)=O(x(\log x)^{-3D})$. Further we consider the equi-distribution property

$$(\mathcal{C}):\sum_{q\leqq x^{1/2}(\log x)^{-B}}\max_{y\leqq x}\max_{(q,l)=1}|E(y;q,l;f)|=O(x(\log x)^{-A}),$$

where

$$E(y;q,l;f)=\sum_{\substack{n\equiv l \pmod q\\ n\leqq y}} f(n)-\varphi(q)^{-1}\sum_{\substack{(n,q)=1\\ n\leqq y}} f(n),$$

$\varphi(q)$ being the Euler function. In the above it is understood that C is a fixed constant and $A, B=B(A)$, D can be taken arbitrarily large and that these are all depending only on f. Then we have

Theorem 1. *Let f and g have the properties $(\mathcal{A}), (\mathcal{B}), (\mathcal{C})$. Then the multiplicative convolution $f*g$ does so.*

2. As for the proof of Theorem 1 we remark the following equality: If $y\leqq x$ and $(q,l)=1$, then

$$\begin{aligned}E(y;q,l;f*g)=&\sum_{\substack{(u,q)=1\\ (\log x)^K\leqq u\leqq x(\log x)^{-K'}}} f(u)E(y/u;q,l\bar{u};g)\\ &+\sum_{\substack{(u,q)=1\\ u<(\log x)^K}} f(u)E(y/u;q,l\bar{u};g)\\ &+\sum_{\substack{(v,q)=1\\ v\leqq(\log x)^{K'}}} g(v)\{E(y/v;q,l\bar{v};f)\\ &\qquad -E(\min(y/v,x(\log x)^{-K});q,l\bar{v};f)\}\\ =&\textstyle\sum_1+\sum_2+\sum_3,\qquad\text{say,}\end{aligned}$$

where $u\bar{u}\equiv 1$, $v\bar{v}\equiv 1 \pmod q$ and K, K' are to be taken appropriately.