96. *Basic Elements over Von Neumann Regular Rings*

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In [4], R. Wiegand and S. Wiegand have shown the following theorem: *Let R be a commutative regular ring. If Spec(R) has no 3-points, then every two-generator faithful R-module has a basic element. The converse holds if R is a Boolean ring.*

They have then asked that (1) for any commutative regular ring, whether the converse holds or not, and (2) whether "two-generator" can be replaced by "finitely generated" or not.

The purpose of this note is to answer (1) in the negative and (2) in the affirmative.

Throughout this note we assume that R is a commutative regular ring with identity 1 and R-modules are unital. We denote by $Q(R)$, $C(R)$ and $X(R)$ the maximal ring of quotients of R, the subring of $Q(R)$ generated by all idempotents in $Q(R)$ over R and the spectrum of R consisting of all prime ideals of R, respectively. For an element a in an R-module A and x in $X(R)$ we denote $a+Ax$ in A/Ax by a_x.

Let A be a finitely generated R-module. An element a in A is called basic in A if, for any x in $X(R)$, the image of a in A_x is part of a minimal generating set of A_x([3]). Since R is regular, as is well known, $R/x \simeq R_x$ and $A/Ax \simeq A_x$ for x in $X(R)$. So a in A is basic iff $a_x \neq 0$ for all $x \in X(R)$.

A point x in a topological space is called an n-point if there are pairwise disjoint open sets $U_1, \cdots, U_n$ such that $x \in U_i - U_i^-$ for $i=1, \cdots, n$, where U_i^- denotes the closure of U_i ([2]).

Lemma 1. *Let $e_1, \cdots, e_n$ be idempotents in R. Then there are orthogonal idempotents $f_1, \cdots, f_m$ in R such that $Re_1 + \cdots + Re_n = Rf_1 + \cdots + Rf_m$, and $e_i f_j = 0$ or f_j any i and j.*

Lemma 2 ([1]). *For x in $X(R)$,*

(i) *in case x is a non-isolated point, it is an n-point iff $[C(R)_x : R_x] \geq n$, the rank of $C(R)_x$ over R_x, and*

(ii) *if x is an isolated point, we have $[C(R)_x : R_x] = 1$.*

Lemma 3. *Let A be a finitely generated R-module. If, for any x in $X(R)$, there is a neighborhood N of x and an element a in A such that $a_z \neq 0$ for all z in N, then A has a basic element.*

Lemma 4. *If a finitely generated R-module A is faithful, then the factor module of A by its singular submodule is also faithful.*