95. *Solution of R. Telgársky's Problem**)

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1. Introduction. In [4], R. Telgársky showed that a paracompact space X has a closure-preserving cover by compact sets if X has two order locally finite covers $\{U_\alpha : \alpha \in A\}$ and $\{C_\alpha : \alpha \in A\}$ such that C_α is compact and U_α is an open neighborhood of C_α for each $\alpha \in A$. Order locally finite covers were introduced by Y. Katuta [2]. In the same paper [4], R. Telgársky showed that a paracompact space with two order locally finite covers which are described above is totally paracompact and that a paracompact space with a closure-preserving cover by finite sets is totally paracompact. In these connections, he raised the question of whether or not a paracompact space with a closure-preserving cover by compact sets is totally paracompact ([4] Problem 2). In the present paper, we shall give an affirmative answer to this problem.

All spaces are assumed to be Hausdorff spaces. N denotes the set of all natural numbers.

A space X is said to be *totally paracompact* [1] if each open basis of X contains a locally finite cover of X. A family $\mathfrak{F}$ of subsets of X is said to be *σ-closure-preserving* if $\mathfrak{F}$ is the countable union of families $\{\mathfrak{F}_n\}_{n=1}^{\infty}$ such that $\mathfrak{F}_n$ is closure-preserving for each $n \in N$.

Theorem 1. *If X is a paracompact space with a σ-closure-preserving cover by compact sets, then X is totally paracompact.*

Corollary 2. *If X is a paracompact space with a closure-preserving cover by compact sets, then X is totally paracompact.*

Corollary 2 is an immediate consequence of Theorem 1.

2. Proof of Theorem 1. When $\mathfrak{U}$ is a family of subsets of a space X, let $\mathfrak{U}^{\#} = \cup\{U : U \in \mathfrak{U}\}$. Let $\mathfrak{F}$ be a closure-preserving family consisting of compact sets of a space X. For each $x \in X$, $K(x)$ is defined to be $X - \cup\{F \in \mathfrak{F} : x \notin F\}$.

When A is a closed subset of X, let

$M_{\mathfrak{F}}(A) = \{x : x \in \mathfrak{F}^{\#} \cap A,\ K(x)$ is not properly contained in any $K(x')$ for $x' \in A\}$.

We need two lemmas to prove Theorem 1.

Lemma 3 (Potoczny [3]). *Let A be a closed subset of a space X*

*) Dedicated to Professor Kiiti Morita for his 60th birthday.