92. *Random Functions in Fourier Restriction Algebras*

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We denote by $A(R)$ the Fourier algebra on the real line R. The norm of $\hat{h}$ in $A(R)$ is

$$\|h\|_1=\frac{1}{2\pi}\int_{\hat{R}}|h(r)|\,dr.$$

For a closed subset E of R, set

$$A(E)=\{g|E : g\in A(R)\},$$
$$\|f\|_{A(E)}=\inf\{\|g\|_{A(R)} : g\in A(R),\ g|E=f\} \qquad (f\in A(E)).$$

Let $E_k=\{x_m^{(k)} : m_k\leqq m<m_k+n_k\}$ $(k=1,2,\cdots)$ be pairwise disjoint finite subsets of R each of which consists of n_k points, where $m_1=0$ and $m_k+n_1=n_2+\cdots+n_{k-1}$ $(k\geqq 2)$. Suppose $x_0\notin\bigcup_{k=1}^\infty E_k$ and $\{E_k\}$ converges to x_0. Put

$$E=\bigcup_{k=1}^\infty E_k\cup\{x_0\}.$$

Let $\{c_k\}$ be a sequence of complex numbers and let $\{\varepsilon_m\}$ be the Rademacher sequence. We define a random function $f=f_\omega$ on E by

$$\begin{cases} f(x_m^{(k)})=\varepsilon_m(\omega)c_k & (k=1,2,\cdots,m_k\leqq m<m_k+n_k)\\ f(x_0)=0. \end{cases}$$

We investigate the condition for the function f to belong to $A(E)$. By using Rudin-Shapiro polynomials, we see that if each E_k is an arithmetic progression and $\{c_k\sqrt{n_k}\}$ does not converge to zero, then there exists a function $f\notin A(E)$. The following Theorem asserts that it holds almost surely. This is based on the same idea as Paley-Zygmund theorem, but we use the estimate of the L^1-norm of random trigonometric polynomials which is due to Uchiyama.

Theorem. *Suppose each E_k is an arithmetic progression. If $\{c_k\sqrt{n_k}\}$ does not converge to zero, then $f\notin A(E)$ a.s.*

Proof. Put

$$x_m^{(k)}=a_k+mb_k \qquad (k=1,2,\cdots,m_k\leqq m<m_k+n_k).$$

For each k, let v_k be the function in $L^1(\hat{R})$ such that

$$\hat{v}_k(x)=\hat{K}_\lambda(x-\{a_k+(m_k+p_k)b_k\}) \qquad (x\in R),$$

where $p_k=[n_k/2]$, $\lambda=p_kb_k$ and

$$\hat{K}_\lambda(y)=\max\left(1-\frac{|y|}{\lambda},0\right) \qquad (y\in R).$$

If $h\in L^1(\hat{R})$ and $\hat{h}=f$ on E_k, then