114. On Holomorphically induced Representations of Exponential Groups

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(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 12, 1976)

The aim of this note is to generalize to the case of exponential groups the results announced in [2] on holomorphically induced representations of split solvable Lie groups.

1. Let $G=\exp \mathfrak{g}$ be an exponential group (for the definition, see [6] for example) with Lie algebra $\mathfrak{g}$, f a linear form on $\mathfrak{g}$, $\mathfrak{h}$ a positive polarization of $\mathfrak{g}$ at f, $\rho(f, \mathfrak{h})$ the holomorphically induced representation of G constructed from $\mathfrak{h}$ and let $\mathcal{H}(f, \mathfrak{h})$ be the space of $\rho(f, \mathfrak{h})$ [1].

In this note, we find a necessary and sufficient condition on $(f, \mathfrak{h})$ for the non-vanishing of $\mathcal{H}(f, \mathfrak{h})$. We then show that $\rho(f, \mathfrak{h})$ $(\neq 0)$ is irreducible if and only if the Pukanszky condition is satisfied, and that in this case $\rho(f, \mathfrak{h})$ is independent of $\mathfrak{h}$. For reducible $\rho(f, \mathfrak{h})$, we describe its decomposition into irreducible components.

The details will appear elsewhere.

2. The triple $(\mathfrak{k}, j, \rho)$ consisting of an exponential Lie algebra $\mathfrak{k}$, a linear operator j and an alternating bilinear form ρ on $\mathfrak{k}$ is called an exponential Kähler algebra if it has the following properties:

a) $j^2=-1$, b) $[jX, jY]=j[jX, Y]+j[X, jY]+[X, Y]$,

c) $\rho(jX, jY)=\rho(X, Y)$, d) $\rho(jX, X)>0$ for $X\neq 0$,

e) $\rho([X, Y], Z)+\rho([Y, Z], X)+\rho([Z, X], Y)=0$.

If, in addition to these properties, there is an linear form ω on $\mathfrak{k}$ such that $\rho(X, Y)=\omega([X, Y])$ for any $X, Y \in \mathfrak{k}$, the triple $(\mathfrak{k}, j, \omega)$ is called an exponential j-algebra. By abuse of language we often call the exponential Lie algebra $\mathfrak{k}$ an exponential Kähler algebra or an exponential j-algebra.

We generalize the structure theorem of a normal j-algebra [4] (resp. a normal Kähler algebra [3]) to an exponential j-algebra (resp. an exponential Kähler algebra).

Theorem 1. *Let* $(\mathfrak{k}, j, \omega)$ *be an exponential j-algebra. We define an inner product S on* $\mathfrak{k}$ *by* $S(X, Y)=\omega([jX, Y])$ *for* $X, Y \in \mathfrak{k}$. *Let* $\mathfrak{a}$ *be the orthogonal complement of* $\eta=[\mathfrak{k}, \mathfrak{k}]$ *with respect to the form S.* $\mathfrak{a}$ *is a commutative subalgebra of* $\mathfrak{k}$, $\mathfrak{k}=\mathfrak{a}+\eta$, *and the adjoint representation of* $\mathfrak{a}$ *on* η *is complex diagonarizable. For* $\alpha \in \mathfrak{a}^*$, *we set* $\eta^{\alpha}=\{X \in \eta;\ [A, X]=\alpha(A)X$ *for all* $A \in \mathfrak{a}\}$ *and let* $\{\eta^{\alpha_i}\}$, $1\leq i\leq r$ *be those root spaces* η^{α} *for which* $j(\eta^{\alpha})\subset \mathfrak{a}$. *Then* $\dim \eta^{\alpha_i}=1$ *and* $r=\dim \mathfrak{a}$ *(r is called*