111. *On the Completeness of Modified Wave Operators*

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The purpose of the present paper is to give a brief account of a proof of the completeness of modified wave operators for long-range scattering.1)

Let $\mathcal{H}=L^2(R^N)$, $N\geq 1$ and put

$$(1)\qquad H_1=-\frac{1}{2}\Delta=-\frac{1}{2}\sum_{j=1}^N \partial^2/\partial x_j^2,\qquad H_2=H_1+V,$$

where $V=V(x)\cdot$ denotes the long-range potential satisfying

$$(2)\qquad |\partial^k V(x)|\leq C_k(1+|x|)^{-k-\beta}\qquad \text{with } 1>\beta>1/2,\ C_k>0$$

for $k=0,1,2,\cdots$. Here ∂^k denotes any k-th order partial differentiation in x. Then, H_1 and H_2 are self-adjoint operators in $\mathcal{H}$. For the pair H_1 and H_2, the existence of the modified wave operators

$$(3)\qquad W_D^{\pm}=\operatorname*{s-lim}_{t\to\pm\infty} e^{itH_2}e^{-itH_1-iX(t)},\qquad X(t)=\mathcal{F}^{-1}\left[\int_0^t V(s\xi)ds\cdot\right]\mathcal{F},$$

was proved by Alsholm-Kato [1] and Buslaev-Matveev [2] (cf. also [7]), where $\mathcal{F}$ denotes the Fourier transform in $\mathcal{H}$. Our problem is to prove the completeness of $W_D^{\pm}$. By definition $W_D^{\pm}$ is complete if $\mathcal{R}(W_D^{\pm})=\mathcal{H}_{2,ac}$, where $\mathcal{H}_{2,ac}$ is the absolutely continuous subspace of H_2 and $\mathcal{R}(T)$ denotes the range of an operator T. To prove this, we shall use the stationary modified wave operators $W_\Gamma^{\pm}$ constructed in [6] and the results of Ikebe [4] (or Saitō [9], [10]). (Here and in the sequel, $\Gamma=[a,b]$, $0<a<b<\infty$, is arbitrarily fixed.) For simplicity, we restrict ourselves to considering only W_D^+ in the following, for W_D^- can be dealt with similarly.

We first summarize those results of [6], [7] and [4] which we need in the sequel.

Theorem 1 (cf. [6] and [7]). *Let W_D^+ be as in* (3) *and let W_Γ^+ be the stationary modified wave operator constructed in* [6]. *Then:*

(i) *$W_\Gamma^+=W_D^+E_{1,ac}(\Gamma)$, where $E_{j,ac}$ is the absolutely continuous part of the spectral measure associated with H_j $(j=1,2)$.*

(ii) *For any $x\in\mathcal{X}_1$, $y\in\mathcal{X}_2$ and Borel subsets Δ_1,Δ_2 of Γ,*

$$(4)\qquad (W_\Gamma^+E_{1,ac}(\Delta_1)x,\ E_{2,ac}(\Delta_2)y)_{\mathcal{H}}=\int_{\Delta_1\cap\Delta_2} e_2(\mu;\tilde{x}^+(\mu),y)d\mu.$$

Here $\mathcal{X}_1=\mathcal{F}^{-1}(C_0^\infty(R^N-\{0\}))$ and $\mathcal{X}_2=L_\delta^2(R^N)\equiv L^2(R^N,(1+|x|)^{2\delta}dx)$, δ

1) Recently, Ikebe also proved the completeness of modified wave operators in a way somewhat different from ours (private communication).