135. *Tensor Products of Positive Definite Quadratic Forms*

By Yoshiyuki KITAOKA
Department of Mathematics, Nagoya University

(Communicated by Kenjiro SHODA, M. J. A., Nov. 12, 1976)

Let L, M, N be positive definite quadratic lattices over $\boldsymbol{Z}$. Our aim is to give some affirmative answers for the following two problems:

i) If M, N are indecomposable, then is $M\otimes N$ indecomposable?

ii) If $L\otimes M$ is isometric to $L\otimes N$, then is M isometric to N?

Definitions and notations. By a positive definite quadradic lattice we mean a lattice L of a positive definite quadratic space V over the rational number field $\boldsymbol{Q}$ (rank $L=\dim V$).

Let L be a positive definite quadradic lattice; then $m(L)$ denotes $\min Q(x)$, where Q is the quadratic form of L and x runs over non-zero elements of L, and moreover we call an element x of L a minimal vector of L if $Q(x)=m(L)$. $\mathfrak{m}(L)$ denotes the set of all minimal vectors of L, and $\tilde{L}$ is by definition the submodule of L spanned by all minimal vectors of L.

Let L, M be positive definite quadratic lattices with bilinear forms B_L, B_M respectively. Then the tensor product $L\otimes M$ over $\boldsymbol{Z}$ is a positive definite quadratic lattice with bilinear form B such that $B(x_1\otimes y_1, x_2\otimes y_2)=B_L(x_1, x_2)B_M(y_1, y_2)$ for any $x_i\in L$, $y_i\in M$.

Through this note $Q(x)$, $B(x, y)$ denote quadratic forms and corresponding bilinear forms $(2B(x, y)=Q(x+y)-Q(x)-Q(y))$, and notations and terminologies will be those of O'Meara [2].

§1. Positive definite quadratic lattices of E-type and their properties.

Definition. Let L be a positive definite quadratic lattice. We say that L is of E-type if every minimal vector of $L\otimes M$ is of the form $x\otimes y$ $(x\in L, y\in M)$ for any positive definite quadratic lattice M.

Theorem. (i) *If L_1, L_2 are of E-type*, then $L_1\perp L_2$, $L_1\otimes L_2$ are of E-type.*

(ii) *If L is of E-type and if L_1 is a submodule of L with $m(L_1)=m(L)$, then L_1 is of E-type.*

(iii) *If L is a positive definite quadratic lattice such that either $m(L)\leq 6$ and the scale sL of $L\subseteq Z$, or rank $L\leq 42$, then L is of E-type.*

This is proved in [1].

§2. **Theorem.** *Let L be an indecomposable positive definite*

*) When we say that L is of E-type, L is assumed to be a poitive definite quadratic lattice.