# 134. A Characterization of Cliffordian Semigroups

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Let $S$ be a semigroup. An element $a$ of $S$ is said to be completely regular if there exists an element $x$ in $S$ such that $axa=a$ and $ax=xa$. A semigroup consisting entirely of completely regular elements is said to be completely regular or Cliffordian. A. H. Clifford [1] proved that completely regular semigroups are semilattices of completely simple semigroups and conversely. Recently, M. Yamada [8] has investigated the regular extensions of a Cliffordian semigroup.

In this short note a new characterization will be given for completely regular elements as well as for completely regular semigroups. For another characterization of Cliffordian semigroups, see [4]. $B(a)$ denote the principal bi-ideal of $S$ generated by the element $a$ of $S$. For other notations and terminology we refer to [2].

**Theorem 1.** *An element $a$ of a semigroup $S$ is completely regular if and only if there exists an idempotent element $e$ in $S$ such that*

$$B(a)=B(e). \tag{1}$$

**Proof.** First, let $a$ be a completely regular element of a semigroup $S$. Then there is an element $x$ in $S$ so that $a=axa$ and $ax=xa$. Hence we have

$$B(a)=aSa. \tag{2}$$

Let $e=ax=xa$. Then $e^2=e$ and $B(e)=(ax)S(xa)\subseteq B(a)$. Also we have

$$B(a)=(axa)S(axa)=e(aSa)e\subseteq eSe=B(e), \tag{3}$$

and we conclude that (1) holds true.

Conversely, if we suppose (1) for an element $a$ of $S$, then

$$B(a)=\{a, a^2\}\cup aSa=B(e)=eSe \tag{4}$$

where $e\in E_S$. (4) implies that there is an element $s$ in $S$ such that

$$a=ese. \tag{5}$$

Hence it follows

$$ea=a=ae. \tag{6}$$

On the other hand, (4) implies

$$e=a,\ e=a^2,\ \text{or}\ e=ata, \qquad \text{where } t\in S. \tag{7}$$

If $e=ata$, we obtain that $a=a^2ta=ata^2$. Hence

$$a=a^2t(ata^2)=a^2(tat)a^2, \tag{8}$$

that is, $a\in a^2Sa^2$. This holds in the other two cases, too. This means that $a$ is a completely regular element of $S$ (cf. [7], IV. 1.2).

**Theorem 2.** *A semigroup $S$ is Cliffordian if and only if every*