133. A Characterization of L^2-well Posedness for Iterations of Hyperbolic Mixed Problems of Second Order

By Kôji KUBOTA
Department of Mathematics, Hokkaido University

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§ 1. **Introduction and theorem.** We are concerned with an iterated mixed problem as follows:

$$(\tilde{P}, \tilde{B}_1, \cdots, \tilde{B}_m)\begin{cases}\tilde{P}(x, D)u = f & \text{in } \Omega,\\ \tilde{B}_j(x', D)u = g_j & \text{on } \Gamma, j=1, \cdots, m.\end{cases}$$

Here Ω and Γ are the open half space $\{x=(x', x_n)=(x_0, x'', x_n)\,;\, x_0 \in R^1, x'' \in R^{n-1}, x_n>0\}$ $(n \geqq 2)$ and its boundary respectively, and for covariable (τ, σ, λ) of (x_0, x'', x_n) the principal symbols $\tilde{P}^0(x, \tau, \sigma, \lambda)$, $\tilde{B}_j^0(x', \tau, \sigma, \lambda)$ of $\tilde{P}, \tilde{B}_j$ have the following forms:

$$\tilde{P}^0 = P_1^0 \cdots P_m^0,\ \tilde{B}_1^0 = B_1^0,\ \tilde{B}_2^0 = B_2^0 P_1^0,\ \tilde{B}_3^0 = B_3^0 P_2^0 P_1^0,\ \cdots,\ \tilde{B}_m^0 = B_m^0 P_{m-1}^0 \cdots P_1^0,$$

where $P_j^0, j=1, \cdots, m$ are x_0-hyperbolic homogeneous operators of second order whose normal cones cut by $\tau=1$ don't intersect each other and are bounded surfaces in the (σ, λ) space for every fixed $x \in \Gamma$. Furthermore B_j^0 is a homogeneous boundary differential operator at most of first order such that Γ is noncharacteristic for B_j^0. All the coefficients are assumed to be real and smooth in $\bar{\Omega}$ and to be constant near the infinity (see [2], [3], [8]).

Definition. *The problem $(\tilde{P}, \tilde{B}_1, \cdots, \tilde{B}_m)$ is said to be L^2-well posed if and only if there exist positive constants C and γ_0 such that for every $\gamma \geqq \gamma_0$ and $f \in H_{1,\gamma}(\Omega)$ the problem $(\tilde{P}, \tilde{B}_1, \cdots, \tilde{B}_m)$ with $g_j=0, j=1, \cdots, m$ has a unique solution u in $H_{2m,\gamma}(\Omega)$ satisfying*

$$\gamma \|u\|_{2m-1,\gamma} \leqq C \|f\|_{0,\gamma}. \tag{1.1}$$

(For function spaces see, e.g., [7]).

Now we have

Theorem. *The problem $(\tilde{P}, \tilde{B}_1, \cdots, \tilde{B}_m)$ is L^2-well posed if and only if all the frozen constant coefficients problems $(\tilde{P}^0, \tilde{B}_1^0, \cdots, \tilde{B}_m^0)_{x'}$ at boundary points $x' \in \Gamma$ are "uniformly L^2-well posed", that is, $(\tilde{P}^0, \tilde{B}_1^0, \cdots, \tilde{B}_m^0)_{x'}$ is L^2-well posed for every $x' \in \Gamma$ and the constants C in* (1.1) *with respect to these problems are independent of the parameter x'.*

§ 2. **Outline of the proof.** It is enough to prove the "if" part, because of Theorem 1 and Lemma 2.2 in [1]. Let $\tilde{L}(x', \tau, \sigma)$ and $L_j(x', \tau, \sigma), j=1, \cdots, m$ be the Lopatinskii determinants of $(\tilde{P}^0, \tilde{B}_1^0, \cdots, \tilde{B}_m^0)$ and (P_j^0, B_j^0) respectively. Then it follows from (3.2) and Theorem 1 in [2] respectively that

$$\tilde{L} = L_1 \cdots L_m \cdot (\text{nonzero factor}) \tag{2.1}$$