131. *On a Degenerate Oblique Derivative Problem with Interior Boundary Conditions*

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1. Introduction. In this note we shall give the unique solvability theorem for a degenerate oblique derivative problem with a *complex* parameter, by introducing an extra boundary condition and adding an error term to the original boundary condition. The background is some work of Egorov and Kondrat'ev [4] and Sjöstrand [6]. In the non-degenerate case such theorem was obtained by Agranovič and Višic [2]. As an application of this theorem, we shall state some results on the angular distribution of eigenvalues and the completeness of eigenfunctions of a degenerate oblique derivative problem having an extra boundary condition. In the non-degenerate case such results were obtained by Agmon [1].

Let Ω be a bounded domain in $\boldsymbol{R}^n$ $(n\geqq 3)$ with boundary Γ of class C^∞. $\bar{\Omega}=\Omega\cup\Gamma$ is a C^∞-manifold with boundary. Let a, b and c be real valued C^∞-functions on Γ, $\boldsymbol{n}$ the unit exterior normal to Γ and α a real C^∞-vector field on Γ. We shall consider the following oblique derivative problem: For given functions f and ϕ defined in Ω and on Γ respectively, find a function u in Ω such that

$$(*)\qquad \begin{cases}(\lambda+\Delta)u=f & \text{in } \Omega,\\ \mathscr{B}u\equiv a\dfrac{\partial u}{\partial \boldsymbol{n}}+\alpha u+(b+ic)u\,|_\Gamma=\phi & \text{on } \Gamma.\end{cases}$$

Here $\lambda=re^{i\theta}$ with $r\geqq 0$ and $0<\theta<2\pi$ and $\Delta=\partial^2/\partial x_1^2+\partial^2/\partial x_2^2+\cdots+\partial^2/\partial x_n^2$.

If $a(x)\neq 0$ on Γ, then the problem $(*)$ is *coercive* and the unique solvability theorem was obtained by Agranovič and Višik [2].

If $a(x)$ vanishes at some points of Γ, then the problem $(*)$ is *non-coercive*. Egorov and Kondrat'ev [4] studied the problem $(*)$ under the following assumptions (A) and (B):

(A) The set $\Gamma_0=\{x\in\Gamma\,;\,a(x)=0\}$ is an $(n-2)$-dimensional *regular* submanifold of Γ.

(B) The vector field α is *transversal* to Γ_0.

In the case that $a(x)$ changes signs on Γ, they proved the non-existence and non-regularity theorem for the problem $(*)$ and, by introducing an extra boundary condition and adding an error term to the original boundary condition $\mathscr{B}u=\phi$, they succeeded in getting a problem for which they could obtain the existence and regularity theorem,