129. *On the Sum of the Möbius Function in a Short Segment*

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1. Let $\mu(n)$ be the Möbius function and let

$$M(x)=\sum_{n\leqq x}\mu(n).$$

Then by the familiar device*)

$$\zeta(s)^{-1}=\zeta(s)^{-1}(1-\zeta(s)H(s))^2+2H(s)-\zeta(s)H(s)^2,$$

where

$$H(s)=\sum_{n\leqq y}\mu(n)n^{-s}$$

with certain y, we can prove that there is an absolute constant $\vartheta, 0<\vartheta<1$, such that

$$(1)\qquad M(x+h)-M(x)=o(h) \qquad (\text{as } x\to\infty)$$

uniformly for $h, x\geqq h\geqq x^{\vartheta}$. But it seems that by this method it is very difficult to get a result which corresponds to Huxley's estimate [3] of the discrepancy between consecutive primes.

In this note we indicate very briefly that there is an alternative way to prove such a result. Our result is as follows:

Theorem. (1) *is true, whenever* $\vartheta>7/12$.

2. Now we show only the main steps of our argument.

We have

$$(2)\qquad M(x+h)-M(x)=\frac{1}{2\pi i}\int_l \zeta(s)^{-1}((x+h)^s-x^s)s^{-1}ds+O(x/T),$$

where l is the straight line connecting the points $1-\delta+iT$ and $1-\delta-iT$, T being sufficiently large and $\delta=(\log T)^{-2/3-\varepsilon}$ with arbitrary small positive constant ε. Here we have used Vinogradov's estimate of the zero-free region of $\zeta(s)$. Let

$$\mathcal{D}=\bigcup_{j=0}^{J}\bigcup_{k=-K}^{K}\Delta(j,k),$$

where $J=[(1/2-\delta)\log T]$, $K=[T(\log T)^{-1}]$ and

$$\Delta(j,k)=\{s=\sigma+it\,;\,\sigma_j\leqq\sigma<\sigma_{j+1},\ k(\log T)\leqq t<(k+1)\log T\},$$

σ_j being $1/2+j(\log T)^{-1}$. We divide $\Delta(j,k)$ into two classes (W) and (Y) as follows: When $\sigma_j\leqq 1-\varepsilon$, then $\Delta(j,k)\in(W)$ if and only if $\Delta(j,k)$ contains at least one zero of $\zeta(s)$, and the remaining rectangles go into

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*) In recent literature this kind of modification has been attributed to Gallagher [1], but this seems originally due to Heilbronn [2].