128. *Canonical Forms of Some Systems of Linear Partial Differential Equations*

By Masaaki YOSHIDA
Kobe University

(Communicated by Kunihiko KODAIRA, M. J. A., Nov. 12, 1976)

§ 1. **Introduction.** It is well known that the linear ordinary differential equation of the second order

$$\frac{d^2u}{dx^2}=q_1(x)\frac{du}{dx}+q_2(x)u \tag{1}$$

is transformed by a change of variable

$$u=a(x)v$$

into an equation of canonical form

$$\frac{d^2v}{dx^2}=p(x)v \tag{2}$$

and that the coefficient $p(x)$ is equal to the Schwarzian derivative of the quotient of two linearly independent solutions of (1). Note that the equation of canonical form (2) is characterized by the condition that the Wronskian of two solutions is reduced to a constant.

In this paper, we shall extend these facts to two kinds of completely integrable systems of partial differential equations:

$$\frac{\partial^2 u}{\partial x_j \partial x_k}=\sum_{\alpha=1}^{n} q_{jk}^{\alpha}(x)\frac{\partial u}{\partial x_\alpha}+q_{jk}^{0}(x)u \qquad (1\leq j,\ k\leq n) \tag{E}$$

and

$$\begin{cases} \dfrac{\partial u}{\partial x_j}=a_j(x)\dfrac{\partial u}{\partial x_j}+b_j(x)u & (1\leq j\leq n) \\ \dfrac{\partial^2 u}{\partial x_j\partial x_k}=c_{jk}(x)\dfrac{\partial u}{\partial x_{i_0}}+d_{jk}(x)u & (1\leq j,\ k\leq n) \end{cases} \tag{E_{i_0}}$$

where all coefficients are supposed to be holomorphic in a domain of $\boldsymbol{C}^n$ and $a_{i_0}(x)\equiv 1$, $b_{i_0}(x)\equiv 0$.

§ 2. **Canonical form of system (*E*).** Given $n+1$ functions $u_0, u_1, \cdots, u_n$ holomorphic in $x_1, \cdots, x_n$, we call

$$\det \begin{bmatrix} u_0 & \cdots & u_n \\ \dfrac{\partial u_0}{\partial x_1} & & \dfrac{\partial u_n}{\partial x_1} \\ \vdots & & \vdots \\ \dfrac{\partial u_0}{\partial x_n} & & \dfrac{\partial u_n}{\partial x_n} \end{bmatrix}$$

the Wronskian of $u_0, u_1, \cdots, u_n$ and denote by $W(u_0, u_1, \cdots, u_n)$.