

146. On Algebraic Threefolds of Parabolic Type

By Kenji UENO

Department of Mathematics, Faculty of Science, Kyoto University,
Kyoto, Japan

(Communicated by Kunihiko KODAIRA, M. J. A., Dec. 13, 1976)

§ 1. In the present note all algebraic varieties are assumed to be complete, irreducible and defined over the complex number field \mathbb{C} . A non-singular algebraic variety is called an algebraic manifold.

Let V be an algebraic manifold and we let K_V (resp. Ω_V^p) denote the canonical bundle (resp. the sheaf of germs of holomorphic p -forms) of V . Put

$$\begin{aligned} P_m(V) &= \dim_{\mathbb{C}} H^0(V, \mathcal{O}(mK_V)), & m &= 1, 2, 3, \dots, \\ h^{p,0}(V) &= \dim_{\mathbb{C}} H^0(V, \Omega_V^p), & p &= 1, 2, 3, \dots, \dim V. \end{aligned}$$

It is well-known that these are birational invariants. Further we put

$$\begin{aligned} p_g(V) &= P_1(V), \\ q(V) &= h^{1,0}(V). \end{aligned}$$

$p_g(V)$ (resp. $q(V)$) is called the geometric genus (resp. the irregularity) of V . For a singular algebraic variety V we define

$$\begin{aligned} p_g(V) &= p_g(V^*), \\ q(V) &= q(V^*), \end{aligned}$$

where V^* is a non-singular model of V .

If $P_m(V)$ is positive for a natural number m , we define a rational mapping (the m -th canonical mapping)

$$\begin{array}{ccc} \Phi_{mK} : V & \longrightarrow & \mathbb{P}^N \\ \omega & & \omega \\ z & \longmapsto & (\varphi_0(z) : \varphi_1(z) : \dots : \varphi_N(z)) \end{array}$$

where $\{\varphi_0, \varphi_1, \dots, \varphi_N\}$ is a basis of $H^0(V, \mathcal{O}(mK_V))$. We set $N(V) = \{m > 0 \mid P_m(V) > 0\}$. The Kodaira dimension $\kappa(V)$ of an algebraic manifold V is defined by

$$\kappa(V) = \begin{cases} \max_{m \in N(V)} \dim \Phi_{mK}(V) & \text{if } N(V) \neq \emptyset, \\ -\infty & \text{if } N(V) = \emptyset. \end{cases}$$

The Kodaira dimension $\kappa(V)$ is a birational invariant. Therefore, for a singular algebraic variety V we define

$$\kappa(V) = \kappa(V^*),$$

where V^* is a non-singular model of V .

An algebraic manifold V is called *parabolic type* if $\kappa(V) = 0$. This is equivalent to saying that $P_m(V) \leq 1$ for every positive integer m and there exists a positive integer n such that $P_n(V) = 1$.