11. *The Mean Convergence for Ergodic Theorems*

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1. Introduction. In this paper we deal with the equivalence of various ways of convergence in the ergodic theorems and establish an ergodic theorem for the family of operators. More precisely, it is shown that conditions considered by L. W. Cohen and W. F. Eberlein are equivalent in some sense. This result is applied to get a mean ergodic theorem for families of commuting operators. The details will be published elsewhere.

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2. Preliminaries. We call a matrix (a_{ni}) satisfying the condition (E) if the matrix satisfies the following conditions

(i) $\lim_{n\to\infty} a_{ni}=0$ $(i=0,1,2,\cdots)$,

(ii) $\lim_{n\to\infty} \sum_{i=0}^{\infty} a_{ni}=1$,

(iii) $\sum_{i=0}^{\infty} |a_{ni}|\leqq K$ $(n=1,2,3,\cdots)$,

(iv) $\lim_{k\to\infty} \sum_{i=k}^{\infty} |a_{ni+1}-a_{ni}|=0$ uniformly in n.

Let T be a bounded linear mapping on a Banach space B such that $\|T^n\|\leqq A$. Then W. Cohen showed that this condition (E) is sufficient condition for the following to have. If $V_n x=\sum_{i=0}^{\infty} a_{ni}T^i x$ is sequentially compact, then the sequence converges strongly to an element $x_0 \in B$ and $Tx_0=x_0$.

Let X be a locally convex space and T a continuous linear mapping of X to X. Let $V_n(T)=\sum_{i=0}^{\infty} a_{ni}T^i$, where (a_{ni}) is a matrix that $V_n(T)$ is well defined as a continuous linear mapping of X to X. Then we call $V_n(T)$ satisfying the condition (E_1) if

(i) $\lim_{n\to\infty} \sum_{i=0}^{\infty} a_{ni}=1$,

(ii) $\lim_{n\to\infty} (I-T)V_n(T)x=0$ for $x\in X$,

(iii) $\{V_n(T): n=1,2,3,\cdots\}$ is equi-continuous.

Throughout this paper we denote by F_T and I_T the set of fixed points of mappings T and T^* respectively, where T^* is an adjoint mapping of T. $\frac{1}{n}\sum_{i=1}^{n} T^i$ is denoted by $M_n(T)$.

Remark 1. If T is a linear mapping on a complete locally convex space X such that the family of mappings $\{T^n: n=1,2,3,\cdots\}$ is equi-continuous, and let a matrix (a_{ni}) satisfy the condition (E), then $V_n(T)=\sum_{i=0}^{\infty} a_{ni}T^i$ satisfies the condition (E_1).