9. *Remarks on Ideals of Bounded Krull Prime Rings*

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1. **Introduction.** Throughout this paper all notations and all terminologies are the same as in [6] and [7]. Let R be a bounded Krull prime ring with the non-empty set of minimal non-zero prime ideals, $M(p)$ say, and let Q be the quotient ring of R. Then $R=\bigcap R_P$ $(P\in M(p))$ and each R_P is a noetherian, local, Asano order in Q. Let F be any right additive topology. We denote by R_F the ring of quotients with respect to F (cf. § 7 of [8]). Let F and F' be right additive topologies of integral right R-ideals. If $R_F=R_{F'}$, then they are said to be *equivalent*.

The aim of this paper is to prove the following theorems.

Theorem A. *Let $P_1,\cdots,P_k\in M(p)$ and let $\bar{I}_i$ be any right R_{P_i}-ideals $(1\leqq i\leqq k)$. Then there exists a unit x in Q such that $xR_{P_i}=\bar{I}_i$ $(1\leqq i\leqq k)$ and $x\in R_{P_j}$ for all $P_j\in M(p)$ with $P_j\neq P_i$.*

Theorem B. *Let I be any right R-ideal and let a be any regular element in I. Then there exists an element b in I such that $I^*=(aR+bR)^*$.*

Theorem C. *Let F be any right additive topology of integral right R-ideals. Then*

(1) *If $F\cap M(p)=\phi$, then $F^*=\{I\mid I^*=R\}$ is a unique maximal element in the set of right additive topologies equivalent to F, and $R_F=R$.*

(2) *If $F\cap M(p)\neq\phi$, then $F^*=\{I\mid I^*\supsetneqq P_1^{n_1}\cdots P_k^{n_k}$, where $P_i\in F\cap M(p)\}$ is a unique maximal element in the set of right additive topologies equivalent to F. If $F(p)=M(p)$, where $F(p)=F\cap M(p)$, then $R_F=Q$, and if $M(p)\supsetneqq F(p)$, then $R_F=\bigcap R_P$ $(P\in M(p)-F(p))$.*

2. **The proofs of Theorems.** (a) First we shall prove Theorem A. To this we let $F(p)=\{P_i\mid 1\leqq i\leqq k\}$ and let $I=\bar{I}_1\cap\cdots\cap\bar{I}_k\cap\bigcap_j R_{P_j}$ $(P_j\in M(p)-F(p))$. Then it is clear that I is a right R-ideal. By Lemma 2.1 of [5] $IR_{P_i}=\bar{I}_i$ and $IR_{P_j}=R_{P_j}$. Let $A=P_1\cap\cdots\cap P_k$. Then there exists a regular element c in Q such that $IR_A=cR_A$ by Lemma 3.3 of [6] and so $IR_{P_i}=cR_{P_i}$ $(1\leqq i\leqq k)$. If $c\in R_{P_j}$ for all $P_j\in M(p)-F(p)$, then c is an element satisfying the assertion. If $c\notin R_{P_j}$ for some $P_j\in M(p)-F(p)$, then there are only finitely many elements $P_{k+1},\cdots,P_{k+l}$ in $M(p)$ such that $c\notin R_{P_{k+j}}$ $(1\leqq j\leqq l)$. Let $B=P_{k+1}\cap\cdots\cap P_{k+l}$. Then it follows that $Q=\varinjlim\,(P_{k+1}R_B)^{-n_1}\cdots(P_{k+l}R_B)^{-n_l}$ by Proposition 1.2,