8. On Embedding Torsion Free Modules into Free Modules*)

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Let R be a ring with identity. A right R-module M is said to be torsion free, if M is isomorphic to a submodule of a direct product of copies of $E(R_R)$, the injective hull of R_R. In [4] the author studied the following problem. What is the condition of a maximal right quotient ring Q under which every finitely generated torsion free right R-module becomes torsionless? Specializing the above problem we shall investigate rings for which every finitely generated torsion free right module is embedded into free right modules. Such a ring will be called *right T.F. ring* in this paper. In section 1 we shall give a characterization of right $T.F.$ rings in the case where Q is right self-injective.

If R is *right* QF-3 i.e., R has a unique minimal faithful right module, then, Q is right QF–3 (Tachikawa [7]), however, the converse does not hold in general. In section 2 it is proved that R is right and left QF–3, if and only if so is Q and Q is torsionless as right and left R-modules.

1. Throughout this paper R is a ring with identity and Q denotes a maximal right quotient ring of R. Let $q \in Q$. Set $(q:R)=\{r \in R;\ rq \in R\}$.

Proposition 1.1. *If Q is right self-injective, the following conditions are equivalent.*

(i) *Every finitely generated R-submodule of Q_R is embedded into a free right R-module.*

(ii) *Q_R is flat and $Q \otimes_R Q \cong Q$ canonically.*

Proof. (ii)$\Rightarrow$(i). This is obtained by the method of [4, Theorem 2].

(i)$\Rightarrow$(ii). Since $qR+R$ is finitely generated R-module, it is isomorphic to a submodule of $\bigoplus_{i=1}^{n} R$, finite direct sum of copies of R_R. Hence there exists $\delta_1, \delta_2, \cdots, \delta_n \in \mathrm{Hom}\,(qR+R_R, R_R)$ such that $\bigcap_{i=1}^{n} \mathrm{Ker}\,\delta_i =0$. Since δ_i is extended to $\bar{\delta}_i \in \mathrm{Hom}\,(Q_Q, Q_Q)$, $i=1,2,\cdots,n$, we can take $a_i \in Q$ so that $\delta_i(x)=a_i x$, $x \in qR+R$. Now, R has an identity.

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