5. Compact Complex Manifolds Containing "Global" Spherical Shells

By Masahide KATO
Rikkyo University, Tokyo

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0. Introduction. Fix an integer $n\geq 2$. For ε, $0<\varepsilon<1$, we put

$$S_\varepsilon=\{z\in C^n : 1-\varepsilon<\|z\|<1+\varepsilon\},$$
$$B_\varepsilon=\{z\in C^n : \|z\|<1+\varepsilon\}, \text{ and}$$
$$\Sigma=\{z\in C^n : \|z\|=1\},$$

where $\|z\|=(\sum_{j=1}^n |z_j|^2)^{1/2}$, $z=(z_j)$.

Let X be a compact complex manifold of dimension n. An open subset N of X is called a *spherical shell* if N is biholomorphic to S_ε for some ε.

Definition 1. A spherical shell N in X is said to be *global* if $X-N$ is connected. Otherwise, N is said to be *local*.

It is clear that, if N is local, then $X-N$ has two connected components. Any complex manifolds contain *local* spherical shells. But *global* spherical shells can be contained in only special types of manifolds.

Before stating the main results, we recall the definition of Hopf manifolds.

Definition 2. A compact complex manifold of dimension n (≥ 2) is called a *Hopf manifold* if its universal covering manifold is biholomorphic to $C^n-\{0\}$. A Hopf manifold is said to be *primary* if its fundamental group is infinite cyclic.

1. Main results. Theorem 1. *Suppose that a compact complex manifold X of dimension n (≥ 2) contains a global spherical shell. Then we can construct a complex analytic family $\pi: \mathfrak{X}\to T=\{t\in C: |t|<1\}$ of small deformations of X such that*

(i) $X=\pi^{-1}(0)$,

(ii) $X_t=\pi^{-1}(t)$ $(t\neq 0)$ *is biholomorphic to a compact complex manifold which is a modification of a primary Hopf manifold at finitely many points.*

Corollary 1. *The fundamental group of X is infinite cyclic. In particular, X is non-Kähler.*

We note that X itself is not always a modification of a Hopf manifold. In fact, if $n=2$, *all compact complex surfaces constructed by M. Inoue in* [2] *and* [3], *which are of Class* VII_0 *with positive second Betti numbers, contain global spherical shells, but none of them is a modi-*