4. On Discontinuous Groups Acting on a Real Hyperbolic Space. II

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0°. Let $G^{(n)}$ be the motion group of a real n-dimensional hyperbolic space $\boldsymbol{H}$. In 1° we apply the two theorems in the preceding note [1] to give explicit fundamental domains and fundamental relations for arithmetic discrete subgroups of $G^{(n)}$ where $4\leqq n\leqq 9$. In 2° we show some examples of discrete subgroups by giving fundamental domains in case $n=3$.

1°. We define an arithmetic group Γ of $G^{(n)}$. Let $\boldsymbol{H}$ be the upper half space $\{\xi={}^t(\xi_1,\cdots,\xi_n)\in\boldsymbol{R}^n\,|\,\xi_n\geqq 0\}$ of $\boldsymbol{R}^n$ with metric form $ds^2=\left(\sum_{j=1}^n d\xi_j^2\right)/\xi_n^2$. Let Q be the matrix of degree $(n+1)$

$$\begin{pmatrix} 1_{n-1} & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

where 1_{n-1} means the unit matrix of degree $n-1$. Let X_Q be a connected component of the hypersurface $\{x={}^t(x_1,\cdots,x_{n+1})\in\boldsymbol{R}^{n+1}\,|\,{}^tx.Q.x=-1\}$ of $\boldsymbol{R}^{n+1}$. Then the motion group $G^{(n)}$ is the subgroup

$$\{g\in GL(n+1,\boldsymbol{R})\,|\,{}^tg.Q.g=Q,\ g(X_Q)=X_Q\}$$

of $GL(n+1,\boldsymbol{R})$. Its action on $\boldsymbol{H}=\boldsymbol{H}^n$ is given by $g.\xi=\eta$ for

$$g=\begin{bmatrix} \sigma & \gamma_1 & \gamma_2 \\ {}^t\delta_1 & \alpha_1 & \alpha_2 \\ {}^t\delta_2 & \alpha_3 & \alpha_4 \end{bmatrix}\in G^{(n)},$$

$\sigma\in M(n-1,\boldsymbol{R})$, γ_i, $\delta_i\in\boldsymbol{R}^{n-1}$ ($i=1$ or 2), $\alpha_i\in\boldsymbol{R}$ $(1\leqq i\leqq 4)$, ${}^t\xi=({}^t\xi',\xi_n)$, $\xi'\in\boldsymbol{R}^{n-1}$ where η is defined by ${}^t\eta=({}^t\eta',\eta_n)$, $\eta'\in\boldsymbol{R}^{n-1}$, $\eta'=\left({}^t\delta_2\xi'+\frac{1}{2}({}^t\xi\xi)\alpha_3+\alpha_4\right)^{-1}\left(\sigma\xi'+\frac{1}{2}({}^t\xi\xi)\gamma_1+\gamma_2\right)$ and $\eta_n=\left({}^t\delta_2\xi'+\frac{1}{2}({}^t\xi\xi)\alpha_3+\alpha_4\right)^{-1}\xi_n$. We denote by $\Gamma^{(n)}$ the group $G^{(n)}\cap SL(n+1,\boldsymbol{Z})$. From now on we assume that $4\leqq n\leqq 9$. We construct a fundamental domain F fit for $\Gamma^{(n)}$. We denote by Γ^∞ the subgroup of $\Gamma=\Gamma^{(n)}$ fixing the point at infinity considered to be contained in $\partial\boldsymbol{H}$ and by Δ the set $\{\xi={}^t(\xi_1,\cdots,\xi_n)\in\boldsymbol{H}\,|\,\xi_1+\xi_2<1,\ \xi_1>\xi_3,\ \xi_2>\xi_3,\ \xi_3>\xi_4>\cdots>\xi_{n-1}>0\}$. Then Δ is a fundamental domain for Γ^∞, namely $\bigcup_{g\in\Gamma^\infty} g\bar{\Delta}=\boldsymbol{H}$ and $g\Delta\cap\Delta=\phi$ for any $g\in\Gamma^\infty-\{e\}$ where e means the unit element of $G^{(n)}$. For each $g\in\Gamma-\Gamma^\infty$ we denote