3. *Abelian Groups and N-Semigroups. II*

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1. Introduction. This note takes its name from the paper [4] by Takayuki Tamura. In that paper Tamura shows the following result:

Theorem 1.1. *Let K be an Abelian group and A be the group of integers under addition. If G is an Abelian extension of A by K with respect to factor system $f: K\times K\to A$, then there exists a factor system g such that*

(i) $g(\alpha,\beta)\geq 0$ *for all* α,β *in* K

(ii) g *is equivalent to* f.

There needs to be a slight change in the proof. Define a new function δ' by $\delta'(\varepsilon)=0$ and $\delta'(\alpha)=\delta(\alpha)$ if $\alpha\neq\varepsilon$. Let $g(\alpha,\beta)=f(\alpha,\beta)+\delta'(\alpha)+\delta'(\beta)-\delta'(\alpha\beta)$.

In his paper Tamura asks if A in Theorem 1.1 can be replaced by an ordered Abelian group. We shall show that A can be replaced by any subgroup of the additive reals. Alternatively we shall show that A can be an Archimedean ordered Abelian group, as an Archimedean ordered Abelian group is isomorphic to a real semigroup.

2. Preliminary results. Let A be a subgroup of the reals under addition. Let G be an Abelian group containing A. Let S be an N-subsemigroup (see [4]) of G which contains $A^+=\{x\in A: x>0\}$ such that G is the quotient group of S. We call A^+ positive cone of A. Let $G=\bigcup_{\xi\in G/A}A_\xi$ be the decomposition of G into cosets modulo A. Let $x\in A_\xi$, some arbitrary coset of G, then $x=bc^{-1}$ for some $b,c\in S$. Let $a\in A^+\subset S$. As S is Archimedean there exists positive integer m and some $d\in S$ such that $cd=a^m$. Thus $xc=b$ implies $xa^m=xcd=bd\in S$. Note that as $x\in A_\xi$ and as $a^m\in A$ we have $xa^m\in A_\xi$ and so $S\cap A_\xi\neq\emptyset$.

Proposition 2.1. *Let A be a subgroup of the reals under addition and G be an Abelian group containing A. Let S be an N-subsemigroup of G which contains A^+. The following are equivalent:*

(i) *G is the quotient group of S.*

(ii) $G=AS$.

(iii) *S intersects each congruence class of G modulo A.*

Proof. We have shown that (i) implies (iii). For any commutative cancellative semigroup T, we let $Q(T)$ denote the quotient group of T. If $G=AS$ then as $A^+\subset S$ we have $A=Q(A^+)\subset Q(S)$ and so $G=AS$