1. On Cauchy Problem for a System of Linear Partial Differential Equations with Constant Coefficients

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1. Introduction. We shall consider the Cauchy problem for a system of partial differential equations for a system of unknown functions $u_\mu = u_\mu(t, x)$ $(\mu=1, \cdots, k)$ of two independent real variables t and x:

$$\partial_t u_\mu = \sum_{\nu=1}^{k} P_{\mu\nu}(\partial_x) u_\nu \qquad (\mu=1, \cdots, k),$$

where $P_{\mu\nu}(\zeta)$ are polynomials in ζ with constant complex coefficients. Using vector-matrix notations we can write for the above system of equations as

$$(1) \qquad \partial_t u^{\downarrow} = \boldsymbol{P}(\partial_x) u^{\downarrow},$$

where $u^{\downarrow} = (u_\mu, \mu \downarrow 1, \cdots, k)$ and $\boldsymbol{P}(\zeta) = (P_{\mu\nu}(\zeta)^{\mu \downarrow 1, \cdots, k}_{\nu \to 1, \cdots, k})$.

Let $\mathcal{F}$ be a linear space of (generalized) complex vector valued functions on $\boldsymbol{R}^1$ such that $\mathcal{S}^k \subset \mathcal{F} \subset \mathcal{S}'^k$,[1] where the topology of the space on the left side of $\subset$ is finer than that of the space on the right side of $\subset$.

The Cauchy problem for the equation (1) is said to be forward $\mathcal{F}$-well posed on the interval $[0, \tau]$ $(\tau > 0)$, if and only if the following two conditions are satisfied.

1) (*Unique existence of the solution*) *For any* $u_0^{\downarrow} \in \mathcal{F}$ there exists a unique $\mathcal{F}$-valued solution $u^{\downarrow} = u^{\downarrow}(t, x)$ of (1) for $t \in [0, \tau]$ with the initial condition $u^{\downarrow}(0, x) = u_0^{\downarrow}(x)$.

2) (*Continuity of solution with respect to the initial value*) If the initial value $u_0^{\downarrow}$ tends to zero in $\mathcal{F}$, then the solution $u^{\downarrow} = u^{\downarrow}(t, x)$ of (1) with the initial value $u^{\downarrow}(0, x) = u_0^{\downarrow}(x)$ also tends to zero in $\mathcal{F}$ uniformly for $t \in [0, \tau]$.

Since the operator $P(\partial_x)$ does not depend on the time variable t, we can easily see that the forward $\mathcal{F}$-well posedness does not depend on $\tau > 0$, hence we can simply use the forward $\mathcal{F}$-well posedness without mentioning the interval $[0, \tau]$.

Making use of the Fourier transform with respect to the space variable x

$$v^{\downarrow}(\xi) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} e^{-i\xi x} u^{\downarrow}(x) dx,$$

1) $u^{\downarrow} \in \mathcal{S}^k$ $(\mathcal{S}'^k)$ means that $u_\mu \in \mathcal{S}$ $(\mathcal{S}')$ for every $\mu=1, \cdots, k$, where $\mathcal{S}$ denotes the set of all rapidly decreasing C^∞ functions on $\boldsymbol{R}^1$ and $\mathcal{S}'$ means the dual space of $\mathcal{S}$.