37. *The Singular Cauchy Problem for Systems whose Characteristic Roots are Non Uniform Multiple*

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(Communicated by Kôsaku YOSIDA, M. J. A., March 12, 1977)

1. Introduction. In [3], applying the method of B. Granoff-D. Ludwig [1], the author studied the singularities of the solutions of the Cauchy problem for a certain system which has a pair of characteristic roots with non uniform multiplicities. It was Duhamel's principle which played a fundamental role in [1] and [3]. The aim of this note is to show that a thorough use of Duhamel's principle enables us to generalize the result of [3] in case more than two characteristic roots are non uniform multiple. Since the calculations become more complicated as the number of the characteristic roots with non uniform multiplicities increases, we only treat the simplest case which three characteristic roots are non uniform multiple. Therefore we must prepare another proof when arbitrary number of characteristic roots are non uniform multiple. However, one may easily conjecture the theorem concerning the general case from this note.

2. Assumptions and result. Let $(t, x)=(t, x_1, \cdots, x_n) \in \boldsymbol{C}^{n+1}$ and $\xi=(\xi_1, \cdots, \xi_n)$ be the covector at $x=(x_1, \cdots, x_n)$.

Consider the system:

$$\mathcal{L}u=\partial u/\partial t+\sum_{\mu=1}^{n} A^{\mu}(t, x)\partial u/\partial x_{\mu}+B(t, x)u=0, \tag{2.1}$$

where A^{μ}, B are $k\times k$-matrices holomorphic in a neighborhood of the origin. We impose a Cauchy data which has a pole on $x_1=0$ and denote this Cauchy problem by (CP).

Now we assume the following assumptions (I) ~ (V).

(I) For any $(t, x)\sim 0$ and $\xi\sim(1, 0, \cdots, 0)$, the matrix $-\sum_{\mu=1}^{n} A^{\mu}(t, x)\xi_{\mu}$ has k (counting multiplicities) eigenvalues $\lambda^l(t, x; \xi)$ $(1\leq l\leq k)$ and the associated eigenvectors $R^l(t, x; \xi)$ $(1\leq l\leq k)$ form a complete set.

(II) For any $(t, x)\sim 0$ and $\xi\sim(1, 0, \cdots, 0)$, each set of eigenvalues $\{\lambda^1, \lambda^l; 4\leq l\leq k\}$, $\{\lambda^2, \lambda^l; 4\leq l\leq k\}$, $\{\lambda^3, \lambda^l; 4\leq l\leq k\}$ is mutually distinct.

(III) λ^l, R^l $(1\leq l\leq 3)$ are holomorphic in $(t, x)\sim 0$, $\xi\sim(1, 0, \cdots, 0)$.

(IV) The Poisson brackets $\{\lambda^i, \lambda^j\}$ $(1\leq i<j\leq 3)$ vanish for $(t, x)\sim 0$, $\xi\sim(1, 0, \cdots, 0)$.

In order to state the assumption (V), we define the phases $F^i(t, x)$ $(1\leq i\leq k)$ and the multi-phases $F^{ij}(t, x; s_1)$ $(1\leq i<j\leq 3)$, $F^{123}(t, x; s_1, s_2)$ by