35. On Certain Ray Class Invariants of Real Quadratic fields

By Takuro SHINTANI
Department of Mathematics, University of Tokyo

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1. In this article we introduce certain ray class invariants of real quadratic fields which are intimately related to the values at $s=1$ of certain L functions of the fields. Then we present a few conjectures on the arithmetic nature of the invariants. Our conjectures are closely related to the third conjecture presented by H. M. Stark in his Nice Congress talk [5]. Assuming suitable additional hypotheses, we prove the conjecture. Unfortunately, the hypotheses are rather restrictive. However, there are several numerical evidences which are in favor of the conjecture even when the hypotheses are not satisfied. The full exposition of the present paper will appear elsewhere.

2. For a pair of *positive* numbers $\omega=(\omega_1, \omega_2)$, we denote by $\Gamma_2(z, \omega)$ the double gamma function introduced and studied by E. W. Barnes in [1]. If we use notations of [3], $\Gamma_2(z, \omega)=F(\omega, z)^{-1}$. Set

$$\Phi(z, \omega)=\frac{\Gamma_2(z, \omega)}{\Gamma_2(\omega_1+\omega_2-z, \omega)}.$$

If ω_1/ω_2 *is irrational,* $\Phi(z, \omega)$ is, as a function of z, characterized by the following properties (1) and (2).

(1) $\Phi(z, \omega)$ is a meromorphic function of z which satisfies the following difference equations:

$$\Phi(z+\omega_1, \omega)=2 \sin\left(\frac{\pi z}{\omega_2}\right)\Phi(z, \omega),$$

$$\Phi(z+\omega_2, \omega)=2 \sin\left(\frac{\pi z}{\omega_1}\right)\Phi(z, \omega).$$

(2) $\Phi\left(\frac{\omega_1+\omega_2}{2}, \omega\right)=1.$

3. Let F be a real quadratic field embedded in the real field $\boldsymbol{R}$. For an integral ideal $\mathfrak{f}$ of F, we denote by $H(\mathfrak{f})$ the group of *narrow* ray classes modulo $\mathfrak{f}$. *We assume that* $\mathfrak{f}$ *satisfies the following conditions* (3) *and* (4).

(3) For any totally positive unit u of F, $u+1 \notin \mathfrak{f}$.

(4) There is no unit u of F such that $u-1 \in \mathfrak{f}$, $u>0$ and $u'<0$, where u' is the conjugate of u.

Take a totally positive integer ν of F such that $\nu+1 \in \mathfrak{f}$. We de-